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1

2 **A model of hoarfrost formation on a cable**

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5

6 **Abstract**

7 A time-dependent numerical model of hoarfrost formation on an overhead cable is
8 presented. The model is aimed at calculating the thickness of a hoarfrost layer using
9 routinely measured meteorological data as input. The growth rate and density of hoarfrost
10 are simulated. This requires continuous calculation of the surface temperature around the
11 cable. The model also simulates the disappearance of hoarfrost by sublimation, melting and
12 dropping off. The feasibility of explaining the occurrence of corona losses on overhead
13 power transmission cables by the modelled hoarfrost thickness is demonstrated by field data.

14

15

16 **Keywords:** hoarfrost; frost; icing; ice accretion; ice growth; power line; corona loss

17

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18 Nomenclature

- 19 C corona loss in electric power transmission, MW
- 20 c_c specific heat of the cable material, J/(kg K)
- 21 c_p specific heat of air, J/(kg K)
- 22 d cable diameter, m
- 23 D iced cylinder diameter, m
- 24 e_a water vapor pressures in air, Pa
- 25 e_s water vapor pressure over ice, Pa
- 26 g gravitational constant, m/s²
- 27 Gr Grashof number
- 28 H mean thickness of the simulated hoarfrost deposit, mm
- 29 h convective transfer coefficient, $h = k_a Nu/D$, W/(m² K)
- 30 I rate of ice formation, kg/(m² s)
- 31 k_a heat conductivity of air, W/(m °C)
- 32 k_i heat conductivity of the simulated hoarfrost deposit, W/(m K)
- 33 L_e latent heat of sublimation of ice at t_s , J/Kg
- 34 n cloudiness, parts of ten
- 35 Nu Nusselt number

- 36 Nu_v Nusselt number in free convection
- 37 Nu_b Nusselt number in forced convection
- 38 p_a atmospheric pressure, Pa
- 39 q_c flux of sensible heat to air, W/m^2
- 40 q_e heat flux due to release of latent heat of sublimation, W/m^2
- 41 q_{eff} effective outgoing long wave radiation flux, W/m^2
- 42 $q_{eff,0}$ outgoing long wave radiation flux at clear skies, W/m^2
- 43 q_i conductive heat flux onto the ice surface through ice, W/m^2
- 44 q_s effective Sun's short wave radiation flux, W/m^2
- 45 Q_J Power due to Joule heating of the cable, W/m
- 46 Q_t Power due to thermal inertia of the cable, W/m
- 47 Re Reynolds number, $\nu D/\nu$
- 48 t_a air temperature, $^{\circ}\text{C}$
- 49 T_a air temperature, K
- 50 t_c cable temperature, $^{\circ}\text{C}$
- 51 T_c cable temperature, K
- 52 t_s ice surface temperature, $^{\circ}\text{C}$
- 53 T_s ice surface temperature, K

54 v wind speed, m/s

55

56 *Greek symbols*

57 ν cinematic viscosity of air, m^2/s

58 σ Stefan-Boltzmann coefficient, $\text{J}/(\text{m}^2\text{K}^4)$

59 ρ density of the forming hoarfrost, kg/m^3

60 ρ_c density of the cable material, kg/m^3

61 ρ_t simulated overall hoarfrost density, kg/m^3

62 ΔT_c decrease in cable temperature during a model time-step, K

63 $\Delta \tau$ time-step in the model, s

64

65

66 **1. Introduction**

67 Hoarfrost forms when water vapour changes directly into solid ice. Under natural
 68 atmospheric conditions the resulting ice deposition consists of loosely spaced needle-like
 69 thin ice crystals (Figure 1). The dimensions of hoarfrost deposits are usually small and the
 70 density of them is very low. Hoarfrost is easily blown off by the wind. Consequently,
 71 hoarfrost alone does not cause ice loads or aerodynamic effects that are significant to
 72 structural safety, unlike rime, glaze and wet snow (Makkonen, 2000).

73



74

75 Figure 1. Hoarfrost on a crabapple showing the typical needle-like microstructure and
76 nonsymmetrical accretion. Photo by S. von Schroeder.

77

78 Nevertheless, hoarfrost causes problems, particularly on overhead cables on which it is
79 related to corona discharges and consequent power losses (Lahti et al. 1997). Corona losses
80 in power transmission are caused also by precipitation, but their magnitude is at its greatest
81 when the electric field at the conductor surface is enhanced by the presence of the ice
82 needles of a hoarfrost deposition. This phenomenon causes electricity transmission losses
83 that are of significant economic value in cold regions (Sollerkvist et al., 2007). Furthermore,
84 on overhead cables for trains and trolleys, hoarfrost causes a corona discharge at the contact

85 with the pantograph, causing excess wear and a light and noise problem (Kamata et al.,
86 2012).

87 During hoarfrost formation in nature the humidity of air is not necessarily high. The
88 outgoing long wave radiation may cool the surface so much that deposition occurs even at a
89 relatively low humidity. When the air is so humid that it is saturated with respect to ice,
90 deposition occurs even without the radiation cooling effect. Thus, hoarfrost may form
91 simultaneously with in-cloud icing. Intensive icing due to accreting droplets may then occur
92 and the portion of the icing rate due to vapour deposition is typically small. However, the
93 rate of hoarfrost formation is approximately proportional to the surface area, whereas the
94 rate of rime icing increases more slowly with increasing object size. Therefore, for ice load
95 modelling on very large objects hoarfrost may need to be taken into account. Moreover, very
96 accurate modelling of rime icing, required by the rotating multi-cylinder method to measure
97 cloud liquid water and droplet size, must include vapour deposition (Makkonen, 1992).

98 Hence, there are several applications in which numerical modeling of hoarfrost formation
99 under natural conditions is useful. The best prospect in utilizing hoarfrost modeling is in the
100 significant savings to be achieved if hoarfrost on cables could be predicted using a weather
101 forecasting model and reduced by controlled Joule heating. Here, a physical-numerical
102 model of hoarfrost is proposed. In contrast to some other models of frost formation on
103 planar surfaces and in industrial applications (Schneider, 1977; Hayashi et al., 1977; Saito et
104 al., 1984; Seki et al., 1984; Östin and Andersson, 1991; Raju and Sherif, 1992; Mago and
105 Sherif, 2005; Kandula, 2011), this model is developed specifically for simulating hoarfrost
106 thickness on a cable in a natural outdoors environment.

107

108 **2. The model**

109 The foundation of this hoarfrost formation model is the cylinder icing model by Makkonen
110 (1984), discussed also in Poots (1996) and Makkonen (2000). The changes made to the icing
111 model for this study on hoarfrost are:

- 112 1. A sub-program simulating vapor deposition rate and hoarfrost density is included, as will
113 be explained below.
- 114 2. Free convection is taken into account in the heat balance because hoarfrost may form
115 when there is no wind.
- 116 3. The outgoing long wave radiation and Sun's direct radiation are included in the heat
117 balance since hoarfrost may form during clear skies.
- 118 4. The heat balance is calculated separately on the windward side and the lee side, and on
119 the upper side and lower side of the object.
- 120 5. Heat transfer from the conductor to the surface of ice is modeled. This allows including
121 the effects of the Joule heating and cable thermal inertia.
- 122 6. The heat balance terms related to droplet impingement are neglected since this model is
123 for simulating hoarfrost only. A necessary input then is the humidity of air.
- 124 7. Ice disappearance is modeled by including evaporation and melting, as well as a criterion
125 of ice release by shedding. This makes long-term continuous modeling possible.

126 The model simulates the mean rate of ice accretion, i.e. the deposition rate, and the mean
127 thickness of the accretion around the cable. They are based on modeling icing at four
128 sections around a horizontally oriented cable. This requires solving numerically the heat
129 balance and surface temperature of these sections, as discussed in the following.

130

131 2.1 Deposition

132 The vapor deposition rate is I calculated by eq. (1). When I is negative, sublimation of ice
 133 occurs (when the simulation shows ice on the section surface).

$$134 \quad I = \frac{0.62}{c_p P_a} h (e_s - e_a) \quad (1)$$

135 Here c_p is the specific heat of air, p_a the atmospheric pressure, h the convective transfer
 136 coefficient and e_s and e_a the water vapor pressures over ice and in air, respectively. The
 137 convective transfer coefficient h depends mainly on wind speed and cable diameter as well
 138 as surface roughness, see below and Makkonen (1985). The vapor water pressure in air, e_a ,
 139 is an input parameter for the modeling. It is typically obtained by measuring the relative
 140 humidity and temperature in air. The equilibrium water vapor pressure over ice, e_s , depends
 141 on the surface temperature of the ice, t_s , the modeling of which is, therefore, a critical issue
 142 here. Moreover, eq. (1) only applies when $t_s < 0$ °C.

143

144 2.2 The heat balance

145 The surface temperature of the ice, T_s , is solved from the heat balance of the section of the
 146 cable by numerical iteration.

147 With the assumptions explained above the heat balance in the model is

$$148 \quad q_e + q_s + q_i = q_c + q_{eff} \quad (2)$$

149 where the heat flux terms are explained in the nomenclature and discussed in the following.

150

151 2.2.1 Convective heat transfer

152 The heat flux due to release of latent heat of phase change from vapor to ice is

153
$$q_e = L_e I \quad (3)$$

154 where L_e is the latent heat of sublimation of ice at t_s .

155 The heat flux due to convective heat transfer to air is

156
$$q_c = h(T_s - T_a) \quad (4)$$

157 Both q_e and q_c depend on the convective heat transfer coefficient

158
$$h = k_a Nu/D \quad (5)$$

159 which is calculated separately on the upper side and lower side of the cable, and in the
 160 presence of wind separately on the windward and the leeward surfaces. Here k_a is the heat
 161 conductivity of air, D is the iced cylinder diameter and Nu is the Nusselt number.

162 In free convection the Nusselt number is calculated by Sparrow and Stretton (1985)

163
$$Nu_v = 0.395 Gr^{0.25} \quad (6)$$

164 where Gr is the Grashof number

165
$$Gr = \frac{gD^3 |T_s - T_a|}{\nu^2 T_a} \quad (7)$$

166 Here g is the gravitational constant, ν is the kinematic viscosity of air, T_s the ice surface

167 temperature and T_a the air temperature. Free convection, calculated by eq. (6), is assumed to
 168 affect the upper side of the cable when $T_s < T_a$ (downward flow) and the lower side of it
 169 when $T_s > T_a$ (upward flow).

170 In forced convection, at a wind speed v , the Nusselt number is calculated by the Reynolds
 171 number $Re = vD/\nu$ as

$$172 \quad Nu_b = aRe^{0.85} \quad (8)$$

173 In eq. (8) the constant a is 0.032 for the windward side and 0.007 for the lee side. This
 174 equation gives allowance to the roughness of the ice surface (Makkonen, 1984). In the
 175 simulations the bigger of Nu_v and Nu_b is applied for each section of the iced cable surface
 176 separately.

177

178 2.2.2 Radiation heat transfer

179 Because hoarfrost forms also at clear skies, a treatment of radiation in the heat balance is
 180 necessary. In fact, the outgoing long-wave radiation is the main driving force for hoarfrost
 181 formation as it cools the surface and thus lowers e_s in eq. (1).

182 The outgoing long wave radiation flux is calculated by eq. (9)

$$183 \quad q_{eff,0} = T_s^4 - T_a^4 \left(0.58 + 0.044\sqrt{e_a} \right) \quad (9)$$

184 where σ is the Stefan-Boltzmann coefficient. Equation (9) is applied on the upper section of
 185 the cable only, which includes the assumption that the radiation fluxes from the ground and
 186 from the lower side of the cable cancel out. In other words, the iced cable emits long-wave
 187 radiation into the half-space above. A correction for cloudiness is made by

$$188 \quad q_{eff} = (1 - 0.08n) q_{eff,0} \quad (10)$$

189 where n is cloudiness in parts of ten. These and other long-wave radiation flux
 190 parameterizations from ice and snow surfaces are discussed in detail in e.g. Gray and Male
 191 (1981) and Sedlar and Hock (2009).

192 Short-wave radiation by the Sun, q_S , is typically a small term here, because the formation of
 193 hoarfrost usually occurs during dark hours. Moreover, the reflection coefficient for short-
 194 wave radiation from a hoarfrost surface is similar to that of fresh snow, i.e. more than 90%
 195 (Gray and Male, 1981, p. 34) and that from a non-iced aluminum cable is similar. However,
 196 q_S can be included in the modeling when the data are available from measurements at
 197 weather stations. Moreover, when direct radiation measurements are unavailable, q_S may be
 198 parameterized based on the latitude, time of year, time of day and corrections based on the
 199 cloudiness and humidity of air (Gray and Male, 1981).

200

201 2.2.3 Conductive heat transfer

202 The conductive heat flux to the surface of the hoarfrost layer q_i equals the heating power
 203 originating from the cable divided by the surface area. An electric cable emits heat in
 204 stationary conditions as the Joule heating Q_J . Additionally, in varying conditions, the
 205 thermal inertia of the cable may produce a heating/cooling power, Q_t . Thus,

$$206 \quad q_i = (Q_J + Q_t) / (\pi D) \quad (11)$$

207 In the modeling, the Joule heating, Q_J , is given as input based on the known cable electric
 208 resistance and the instantaneous current.

209 The instantaneous power by the thermal inertia, Q_t , in eq. (11) is calculated based on the
 210 changes in the atmospheric conditions and the electric current within a time-step of the
 211 model. This requires simulation of the temperature of the cable, which is calculated based on
 212 the cylindrical geometry as

$$213 \quad T_c = T_s + D \ln(D/d) q_i / (2k_i) \quad (12)$$

214 where d is the cable diameter and k_i is the heat conductivity of the hoarfrost estimated by

$$215 \quad k_i = 0.0242 + 0.0002\rho_i + 2.54 \cdot 10^{-6} \cdot \rho_i^2 \quad (13)$$

216 Here, ρ_i is the simulated overall hoarfrost density (see 2.4). Equation (13) is a modified
 217 extension of an empirical equation by Yonko and Sepsy (1967). The power per unit length
 218 of the cable caused by thermal inertia within a time-step $\Delta\tau$ of the model is then calculated
 219 as

$$220 \quad Q_t = \pi (d/2)^2 \rho_c c_c \Delta T_c / \Delta\tau \quad (14)$$

221 where ρ_c and c_c are the density and the specific heat of the cable material respectively and
 222 ΔT_c is the calculated decrease in the cable temperature during a time step of $\Delta\tau$.

223

224 2.4 Hoarfrost density

225 The density of the forming hoarfrost, ρ , is calculated based on the surface temperature, t_s , of
 226 that side of the cylinder where the rate of ice formation, I , is the highest. This is justified by
 227 test runs showing that the accretion is typically quite unsymmetrical with insignificant icing
 228 on the other sections. The hoarfrost density is calculated by eq. (15) presented in Hayashi et

229 al. (1977). This equation is based on experimental data in the range $-25\text{ }^{\circ}\text{C} < t_s < 0\text{ }^{\circ}\text{C}$ and 2
230 $\text{m/s} < v < 6\text{ m/s}$.

$$231 \quad \rho = 650 e^{0.227t_s} \quad (15)$$

232 The density of the overall accretion ρ_t and the thickness of the ice layer are calculated
233 cumulatively after each time-step as explained in Makkonen (1984). When there is
234 disappearance of ice, the density of the lost ice is taken to be that of the overall density.

235

236 **3. Running the model**

237 The model calculates the heat balance and the rate of hoarfrost formation separately on the
238 windward side and the lee side and on the upper side and lower side of the object. The
239 output of the model is the mean thickness of the simulated hoarfrost around the cable, H .
240 Disappearance of hoarfrost by sublimation occurs in the model when eq. (1) gives negative
241 values and ice exists on the cable. In addition, when the modeled cable temperature, t_c (see
242 eq. (12), reaches $0\text{ }^{\circ}\text{C}$, all ice is assumed to shed or melt. No icing is modeled as long as $t_c \geq$
243 $0\text{ }^{\circ}\text{C}$. Shedding of the hoarfrost deposit due to wind or cable dynamics alone is not
244 considered.

245 The model is programmed in Visual Basic. The input for simulations can be given manually
246 or be read directly from meteorological observation data files. In the latter case, the output is
247 given by the model at intervals which correspond to the input. This is three hours for
248 synoptic meteorological data. However, the internal calculation time-step of the model is
249 much shorter for better accuracy. It is 10 seconds for the first 10 minutes of icing and 360
250 seconds after that. Test with the model showed that time-steps shorter than this do not

251 change the output within the desired resolution.

252 The simulation of hoarfrost formation on a power line cable by the model requires as input

253 • Cable diameter

254 • Cable specific heat capacity

255 • Joule heating by the cable (when existent)

256 • Wind speed

257 • Air temperature

258 • Humidity in air

259 • Cloudiness

260 • Sun's radiation (when significant)

261

262 **4. Results and discussion**

263 Test runs using weather data in Finland showed that the model typically predicts a hoarfrost

264 thickness of 0–15 mm, which corresponds to observations in nature. The results show

265 systematic variations according to the time of day, as one would expect (Figure 2). The

266 modeled hoarfrost cover typically forms in the night and disappears during the day.

267 In order to evaluate the sensitivity of the model to the meteorological parameters, it was run

268 by changing one input variable at a time keeping the other input conditions fixed. The

269 relationships revealed correspond well to what is qualitatively observed (Kamata et al, 2012)

270 and, while complex, provide a deeper understanding of hoarfrost formation. The results of

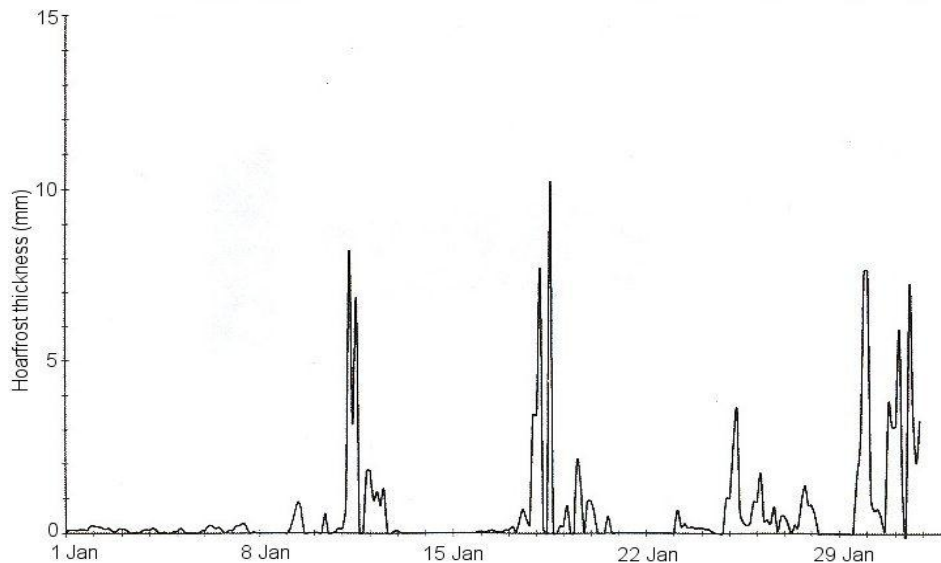
271 the model for the hoarfrost thickness H are quite sensitive to cloudiness, for example. This is

272 shown as in Figure 3. Wind induced convection tends to prevent the ice surface temperature

273 from falling well below that of air, thus resulting in a hoarfrost deposit with a lower density.

274 Therefore, H increases with decreasing wind speed as shown in Fig. 3.

275



276

277 Figure 2. Example of a model output for hoarfrost thickness on a power line cable when
 278 using synoptic weather data (Kajaani, Finland, January 1994).

279

280 The dependence of the modeled hoarfrost thickness H on the relative humidity in air is
 281 shown in Fig. 4. This relationship is similar to that of the mass of sublimated ice. In contrast,
 282 a thicker hoarfrost layer at a colder temperature, also shown in Fig. 4, is entirely due to the
 283 effect of temperature on the density of the hoarfrost, as the modeled ice mass increases with
 284 increasing air temperature at a fixed relative humidity.

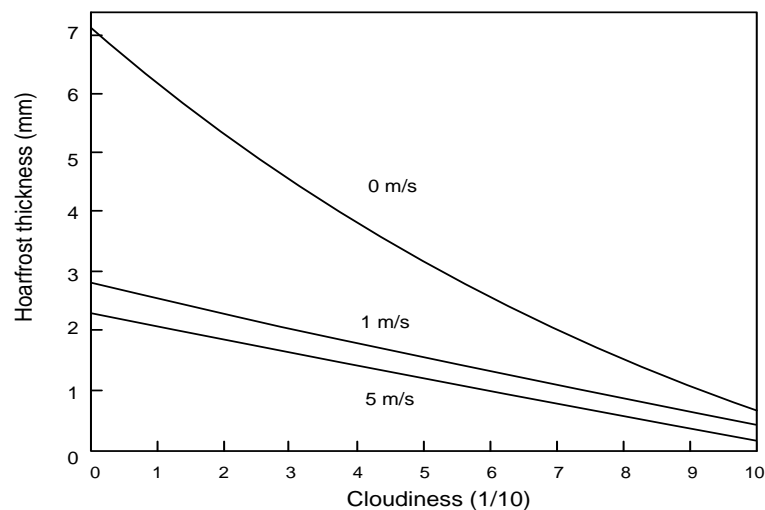
285 The model results critically depend on the input for the humidity in air as shown in Figure 4.

286 In nature, vapor deposition often occurs in air that is supersaturated with respect to ice. The

287 degree of super-saturation then controls the growth rate of hoarfrost. However, the degree of

288 supersaturation is not revealed at all by routine measured weather station data. This is
289 because when hoarfrost forms, ice deposits on the humidity probes as well (Makkonen and
290 Laakso, 2005). Consequently, the conventional humidity probes show a humidity reading
291 that is locked at the saturation value with respect to ice, even when the true value is well
292 above it (Makkonen and Laakso, 2005). This fundamental problem in the measurements
293 removes, from the data, all real variation in the air humidity under supersaturated conditions.
294 It thus seriously hampers detecting the correlation between the modeled and real hoarfrost
295 thickness.

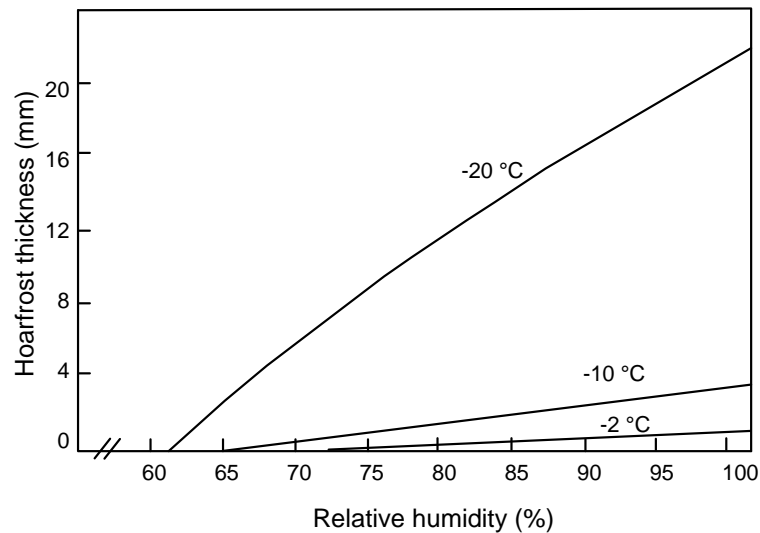
296



297

298 Figure 3. Modeled hoarfrost thickness vs. cloudiness at three wind speeds in a 10-hour
299 simulation on a 50 mm diameter cable at air temperature of -10°C and relative humidity of
300 90%.

301



302

303 Figure 4. Modeled hoarfrost thickness vs. relative humidity at three air temperatures in a 10-
 304 hour simulation on a 50 mm diameter cable at a wind speed of 1 m/s and clear sky.

305

306 In future applications, humidity measurements made by a special sensor that provides
 307 correct values also at supersaturated conditions should be utilized (Makkonen and Laakso,
 308 2005). Until then, it is not possible to fully verify the hoarfrost model quantitatively by
 309 direct observations of hoarfrost thickness in the field. Unfortunately, the model cannot be
 310 tested in a laboratory either, because the main driving force of hoarfrost formation, the
 311 effective outgoing long-wave radiation, is non-existent indoors. However, it is possible to
 312 test the model in its most important application directly, as discussed in the following.

313 The feasibility of the model in detecting the corona loss in power transmission was tested in
 314 large scale. Data for the corona loss in the 360 km long 400kV overhead power line
 315 Alapitkä–Petäjälampi in central Finland were used for the purpose. These power loss data
 316 were measured by a power utility IVO Voimansiirto Oy by a method discussed by
 317 Sollerqvist et al. (2007). The input for the hoarfrost model was obtained from four weather
 318 stations operated by the Finnish Meteorological Institute along the route of the power line.

319 The comparison between the modeled hoarfrost and the measured corona loss was made for
 320 the period 1st January–30thApril 1994. Hoarfrost thickness was modeled at three-hour
 321 intervals for this period at each weather station. A weighted average of these values was
 322 considered to represent the entire line. The weights given for each weather station were
 323 based on their geographical representativeness along the line.

324 The results show a significant correlation between the modeled occurrence of hoarfrost and
 325 the measured corona loss C , particularly when the events with precipitation are removed
 326 from the data. This relationship is shown in Table 1 where $C \leq 0.5$ means “insignificant
 327 loss” $0.5 < C < 2$ “moderate loss” and $C \geq 2$ “significant loss” as classified by the power
 328 utility. Table 1 shows, for example, that out of the cases where the model shows no
 329 hoarfrost, only 3% show moderate or significant corona loss. When the model shows more
 330 than 2 mm hoarfrost thickness the corona loss is moderate or significant in 82% of the cases.
 331 The hoarfrost model, thus, predicts the occurrence of a corona loss in large scale quite well
 332 and is definitely a useful tool in detecting the conditions prone to corona losses.

333

334 Table 1. Modeled mean hoarfrost thickness H (mm) and measured corona loss C (MW) on
 335 the 360 km long overhead power line. The observations are at three-hour intervals over a
 336 period of four months. Cases when a weather station reports precipitation are excluded.

	$H = 0$	$0 < H < 2$	$H \geq 2$
$C \leq 0.5$	186	77	15
$0.5 < C < 2$	5	67	24
$C \geq 2$	1	40	45

337

338 However, there is only a weak correlation ($r = 0.47$) between the modeled thickness of the
339 hoarfrost accretion and the measured quantitative corona loss in these data of 460
340 observations. This is not surprising considering the fundamental problem in the input for air
341 humidity, discussed above. Another problem with the input data is that unobserved local
342 phenomena, such as snow showers and high humidity at river crossings, may have occurred
343 along the power line route between the weather stations.

344 On the other hand, there may also be weaknesses in the model. For example, the possibility
345 that the electric field on the cable affects the hoarfrost formation (Teisseyre and Farzaneh,
346 1990) is not taken into account, because it has been considered unlikely that such an effect
347 would cause a feedback between the corona discharge and ice amount (Zhang et al., 2006).
348 Modeling the hoarfrost thickness as the mean over the cable circumference may not be ideal
349 in this application and options for that will be studied in the future.

350 A particularly interesting result of the modeling is that the sensitivity tests for the entire 360
351 km long power line showed a significant reduction in the frequency of hoarfrost by a rather
352 small increase in the Joule heating. This may provide means to control the hoarfrost
353 formation on power lines in order to reduce corona losses. Another interesting prospect is
354 the short-term forecasting of corona losses by running the hoarfrost model using the output
355 of a numerical high resolution weather prediction model (Nygaard et al., 2011).

356

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361

362

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