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Innovation Incentives in Enterprise Networks

A Game Theoretic Approach

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Abstract

This paper explores the applicability of game theory to the modelling of enterprise networks. Although these networks have traditionally been studied by the qualitative methods of industrial management, the utilisation of game theory seems to provide new tools and solution concepts for studying them. The paper reviews earlier game theoretic studies on inter-firm cooperation and presents a general stepwise pattern for game theoretic modelling of network economy. In addition, the paper constructs a game theoretic model for studying possibilities for creating innovative incentives in an enterprise network.

Inter-firm cooperation is characterised by the interaction of several decision-makers where, on the one hand, the network companies seek joint gains by networking, but, on the other hand, individual companies have their own objectives which may be in partial con ict with those of other companies. Here, game theory provides tools for the formal analysis of situations where multiple decision-makers may have partially con icting interests, but cooperation between them is allowed.

The determination of innovation incentives in enterprise networks is studied through an application of game theoretic modelling. An example from the boatbuilding industry is presented to illustrate the relevance of innovation incentives in enterprise networks. Specifically, three different equilibrium concepts are applied to determine innovation incentives under different circumstances. The proposed model helps award innovations that improve the efficiency of the network. In addition, the efficiency-improving arrangements can be implemented so that none of the network companies has to suffer. Consequently, the enterprise network becomes innovative and the network companies need not fear their own losses when the efficiency-improving arrangements are implemented. The model also helps share the surplus utility gained through the innovation among the companies of the network.

Preface

VTT Industrial Systems / Industrial Management has a long tradition on the research of networked economy. In most of the earlier studies, which have been carried out at the VTT Industrial Management group, the approach to the problem at hand has been qualitative. However, in order to construct theoretically sound tools for enterprise-network management, more formal methods are needed. Hence, VTT Industrial Management group has opened a new field of research, that is *formal decision-making tools for industrial management*.

This paper examines the potential of applying *game theoretic* models and techniques to support decision-making in networked business environment. The general research problem is to apply game theoretic concepts in describing and developing cooperative practices in enterprise networks. A long term objective of the research is to create a game theoretic methodology and tool package for enterprise-network management.

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Toni Jarimo

Contents

1 Introduction

Strategic enterprise networks have provided a new kind of advantage for companies operating in dynamic and competitive fields of business. The earlier hierarchical, buyer-oriented demand supply chains – traditionally found in the US and Japanese auto industries – have had to yield to collaborative multilateral network production. In addition, advancements in information and communication technologies, globalisation of technology-intensive firms, and freedom of movement for researchers have fostered a degree of inter-firm research and development. (Mintzberg et al. 1998, Hyötyläinen 2000)

New modes of industrial activity call for up-to-date models to support decision-making in firms engaged in a network economy. The network companies make their decisions in conditions where, on the one hand, the companies aim at win-win situations with their network partners, but, on the other hand, endeavour to improve their own benefits individually. In consequence, the interests of separate network companies may be partially conflicting, in pursuance of the common objective of global efficiency (Ollus et al. 1998). This view diverges from the usual approach to decision-making, which considers the issue from the perspective of a single decision-maker (see e.g. Clemen 1995).

Traditionally, a network economy has been studied by the qualitative means of industrial management. In this paper, we shall approach enterprise networks from a mathematical perspective. The particular methodology to be used is *game theory*. Game theory is a mathematical means for the formal analysis of the decision-making of multiple decision-makers, called *players*. The players interact with each other in a given framework, the *game*, which is defined specifically in each case. Each player possesses a set of alternative actions, or *strategies*, the execution of which affects the amounts of utility that the players individually obtain. The amounts of utility are measured by the players' *utility functions* (Myerson 1997). It is expected that formal modelling of enterprise networks will provide new tools and solution concepts to aid decision-making concerning network economy.

We postulate that the concepts of game theory fit those of enterprise networks well:

- Companies in the network are the players, with (possibly) partly conflicting interests.

- Companies have their own available strategies, which they play in order to achieve their goals.

- Cooperation between companies can lead to globally increasing utility.

The companies can be, for instance, part of a horizontal network, in a vendor-customer relationship, or even competitors. The applications are many, including pricing of products or services, contract negotiation, and production network management. One of the aims of game theoretic analysis is to *predict the players' behaviour* in situations of (partially) conflicting interests and thus to yield information to support decision-making in a networked business environment. More specifically, game theoretic models aim to bring out optimal strategies for the players, reveal and eliminate possibilities for opportunism, and illustrate causal relationships in interaction between the players, for instance. For practical purposes, we expect that the models can be exploited to identify win-win situations among cooperating companies and thus to improve the competitive advantage of enterprise networks.

The main objective of this paper is to evaluate the applicability of game theoretic models to the management of enterprise networks. We shall find out the prerequisites, limitations, and advantages of game theoretic analysis. In addition, we shall present a framework for the systematic construction of game theoretic models. The modelling process is illustrated by a case study, in which we examine incentive strategies for innovation management in an enterprise network. The pilot company of the case study is the Finnish sailing-yacht builder *Nautor Ltd.*

The structure of this paper is as follows. Section introduces basic concepts of game theory and reviews earlier game theoretic studies concerning enterprise networks. Herein, we also present a general guideline for game theoretic modelling of network cases. Section 2.7 presents a model by which efficiency-improving incentives can be determined and the utility from win-win situations shared among network companies. In Section 3.8 we illustrate the use of the model with a numerical example. Section 4.5 discusses the advantages and disadvantages of our model and summarises the results and conclusions of the paper.

2 Earlier Game Theoretic Studies on Network Economy

2.1 Basic Concepts of Game Theoretic Models

The use of a game theoretic model requires the identification of three elements, namely

1. the *players* of the game
2. the available *strategies* for the players
3. the players' individual *utility functions*, which map the set of all possible strategies to real numbers.

In addition, a game can be characterised according to several case-specific attributes. For instance, the game can be

- *cooperative* or *non-cooperative*
- *of perfect information* or *of imperfect information*.
- *deterministic* or *stochastic*
- *static* or *dynamic*.

Of the above-mentioned, the first pair describes the possibility of interaction between the players. The form of interaction can be e.g. communication, side-payments or readiness to cooperate. The second pair depicts the players' amount of information in the situation. Third, if the game contains uncertainties it is said to be stochastic, otherwise it is deterministic. Lastly, if the players make their decisions successively and the earlier decisions affect the set of available strategies in the forthcoming decision-making, then the game is said to be dynamic. Again, in a static game the players make their decisions simultaneously. (Myerson 1997)

In a cooperative game, the aim is to achieve a *pareto optimal* solution i.e. such strategies for the players in which no strategy can be changed without reducing someone's utility. On the other hand, we may assume that a non-cooperative game with rational players is driven to a *Nash equilibrium*, in which, for each player, it is unprofitable to individually change the strategy from the equilibrium (Nash 1951). The majority of practical cases lie somewhere in between the two extremes of genuine cooperation and total competition.

In a game of imperfect information or in a dynamic game, one player (the *leader*) may have the opportunity to construct *incentives* for other players (the *followers*). The purpose of the incentives is to make the followers choose their strategies optimally for the leader. In addition, such asymmetric situations may create opportunities for cheating or otherwise opportunistic behaviour. Hence, one purpose of game theoretic analysis is to *identify and eliminate possibilities for cheating*. In the following sections, some of the earlier game theoretic studies concerning inter-firm activity are briefly reviewed.

2.2 Cooperation in Research and Development Projects

2.2.1 Repeated Prisoner's Dilemma

In the field of economics, the well-known *Prisoner's Dilemma*[1] has been the subject of interest in numerous studies (see e.g. Jarillo 1993, pp. 138, Nooteboom 1999, Nurmilaakso 2000, pp. 47). The reason for this is that many practical problems reduce to Prisoner's Dilemmas when being modelled as a game.

A widely used example of such a problem is that of a common research project of two separate companies (players). The companies' strategies are either to cooperate or to "free ride" (a concept brought out by e.g. Nooteboom 1999). By free riding we mean that a player tries to benefit from the project without investing in it. If both players cooperate, the research project is a success and both players obtain an equal utility of, say, 10. If one of the players cooperates but the other one free rides, the result is a partial success, in which case the free riding player receives the results of the project for free and derives a utility of 15, whilst the cooperating player has put more effort into the project and, thus, incurs a negative utility -5. Lastly, if both players free ride, there is no gain from the project. Nor have the players contributed to the project, thus giving a zero utility to both players. Figure 1 presents the game in its matrix form.

The research project problem presented in Figure 1 is a Prisoner's Dilemma type of game. The unique Nash equilibrium of this game is that both players choose to free ride (f_1, f_2), which can be seen, for instance, by the fact that, for both players, cooperating is strongly dominated by free riding. However, in playing strategies (c_1, c_2) both players would be better off, that is (f_1, f_2) is not pareto efficient.

[1] For the story behind Prisoner's Dilemma, see e.g. Luce and Raiffa (1957).

		Player 2	
		cooperate (c_2)	free ride (f_2)
Player 1	cooperate (c_1)	(10,10)	(-5,15)
	free ride (f_1)	(15,-5)	(0,0)

Figure 1. Research Project Game in the Matrix Form.

Axelrod (1984) has carried out a fruitful study of *Repeated Prisoner's Dilemma*. In this game, two players play Prisoner's Dilemma repeatedly for an *ex ante* unknown number of turns, always knowing the outcomes of all the previous turns. In the end, the utility for both players is the sum of the utilities of individual rounds. In his book, Axelrod presents the results of a Repeated Prisoner's Dilemma Tournament, in which the competitors' task was to construct an algorithm which would play the game against other algorithms. The winning algorithm appeared to be the simplest one of those sent to the tournament, named "Tit-for-Tat". The run of the algorithm is as follows:

Round 1: Cooperate

Round *k*: Play the same strategy which the opponent played in round $k - 1$.

Axelrod (1984) summarises four properties that make the Tit-for-Tat strategy both robust and successful:

1. It *does not drive the players to conflict* while cooperating as long as the opponent cooperates.

2. It *penalises* the opponent instantly from free riding.

3. It is *forgiving* after a conflict if the opponent is willing to cooperate.

4. Its behaviour is *transparent*, making it easy for the other player to adapt to the pattern of the game.

It is noteworthy that, despite the fourth property, even an action rule particularly aimed at defeating the Tit-for-Tat strategy did not perform any better (see Axelrod 1984). In fact, as Nurmilaakso (2000) suggests, knowing that the opponent is playing Tit-for-Tat, encourages the other player to cooperate. Nurmilaakso's idea is to use *reputation* as a tool to model pledges (i.e. incentives and threats) within enterprise networks.

2.2.2 Nooteboom's Model for Free Riding

Nooteboom (1999, A3.2) presents a more detailed game theoretic study concerning free riding in a joint R&D project. According to Nooteboom, problems tend to arise when it is difficult to *observe* the partners' actual contributions to the project. The situation becomes a Prisoner's Dilemma if bilateral free riding yields a better utility than one-sided cooperation does. This is the case, for instance, if there is a risk of information spillover to a direct competitor.

Several solutions to avoid the problem are suggested. Firstly, if the research topic is complicated or the information produced is tacit, all the partners need to contribute to the project in order to be able to utilise the results achieved through the project. Secondly, some sort of repayment could be constructed. However, the use of such payment requires good measurement of the partners' contributions to the project, which, in many cases, may be impossible to implement. The third solution is the one discussed in Section 2.2.1, i.e. there is hope of joint R&D projects also in the future. Therefore, the partners might be willing to engage in mutual cooperation in order to keep the partnership alive. In addition, even if there is no continuity in sight for the current partnership, the companies might want to sustain their reputation for being a good research ally.

Nooteboom models the characteristics of a joint R&D project – in the concepts of game theory – as follows. The utility function for player i is

$$u_i = -c_i t_i + s_{ij} r_{ji} t_i + (1 - s_{ji}) r_{ij} t_j + p_i t_i t_j, \tag{1}$$

where

- i, j are the indices for partners
- t_i is the amount of contribution to the project from i
- c_i is the unit cost of i's contribution to the project. The main cause for this is the risk that vital information is spilled to a competitor.
- r_{ji} is the unit utility for j of i's contribution
- s_{ij} is the repayment share of $r_{ji} t_i$ from j to i
- p_i denotes the unit utility for i of the partners' teamwork, which is measured as the multiplication $t_i t_j$.

Player 2

Player 1		cooperate $(t_2 = 1)$	defect $(t_2 = 0)$
	cooperate $(t_1 = 1)$	$u_1 = -c_1 + s_{12}r_{21} + (1 - s_{21})r_{12} + p_1$ $u_2 = -c_2 + s_{21}r_{12} + (1 - s_{12})r_{21} + p_2$	$u_1 = -c_1 + s_{12}r_{21}$ $u_2 = r_{21} - s_{12}r_{21}$
	defect $(t_1 = 0)$	$u_1 = r_{12} - s_{21}r_{12}$ $u_2 = -c_2 + s_{21}r_{12}$	$u_1 = 0$ $u_2 = 0$

Figure 2. Game (1) in its Matrix Form.

The game is simplified in such a way that either partner has two alternative strategies:

1. transfer competence to the project, $t_i = 1$

or

2. defect from the project, $t_i = 0$.

Letting available strategies be the aforementioned and the number of players be two, then the game, in its matrix form, is as depicted in Figure 2. Clearly, mutual cooperation becomes a Nash equilibrium if

$$\begin{aligned} & -c_i + s_{ij}r_{ji} + (1 - s_{ji})r_{ij} + p_i > r_{ij} - s_{ji}r_{ij} \\ \Longrightarrow \quad & p_i + s_{ij}r_{ji} > c_i, \end{aligned} \tag{2}$$

i.e. if the unit benefit from teamwork and the repayment share together are greater than the cost of transferring competence into the project. The condition (2) also implies that if there is no benefit from teamwork $(p_i = 0)$, then

$$s_{ij}r_{ji} > c_i \tag{3}$$

is necessary for $(t_i = 1 \ \forall \ i)$ to be the Nash equilibrium. In this case, a positive repayment s_{ij} of the received knowledge is essential. Respectively, if p_i is great enough, more precisely, if

$$p_i > c_i, \tag{4}$$

then there is no need for a repayment at all.

In his study, Nooteboom ends up with several hypotheses concerning the joint R&D project:

H1 Mutual cooperation is likely to take place if the value of the exchanged knowledge (r_{ij}) is high and the risk of knowledge spillover to a competitor (c_i) is low. A high r is usually achieved when the information in question is advanced and complex. Similarly, tacit information, vertical partnership, small number of partners and good monitoring of spillover indicate low c.

H2 When c is high, it is still possible for the partners to arrive at mutual cooperation. However, the requirement is that either condition (2) is fulfilled or there is expectation of future cooperation, whose net present value is greater than the utility gained from one-off defection.

H3 A high c leads to better control of spillover and free riding.

H4 Small firms have few partners and their information is tacit, which drives them to look for more partners, but also makes them interesting candidates to other firms who are looking for partners.

Hypotheses H1-H3 follow directly from the game presented in Figure 2. On hypothesis H4, Nooteboom (1999, pp. 16) gives two reasons for the tacitness of information in small firms:

1. knowledge is more based on craftsmanship, which tends to be tacit
2. production management is more informal, including less written documents or explicit models.

Nooteboom also claims that external relations are of greater value for small firms rather than for large firms, thus motivating small firms to look for more partners. On the other hand, the motivation for a number of private entrepreneurs to be a private entrepreneur is precisely their *independence of other people*.

2.2.3 Optimal Number of Partners According to Nooteboom

The model presented in the previous section is constructed to give an answer to the question whether a company should seek partners or stay independent. The next question – if the answer to the first one is "seek partners" – is then, *how many partners should a company have*? Again, Nooteboom (1999, A3.3) constructs a game, in which the utility function for player i is now

$$u_i = -c_i t_i n_j + r_i t_j + p_i t_i t_j + m_i n_i + v_i t_j n_j, \tag{5}$$

where

- n_i is a binary variable with values $n_i = 1$ when i has multiple partners and $n_i = 0$ when i has a single partner.
- m_i is the benefit for i of having multiple partners. Sources of benefit are e.g. improvement of bargaining position, sharing risk among partners, and multiple sources of learning.
- v_i is the additive benefit for i of i's partner having multiple partners, hence increasing the number of sources of learning.

Other variables are identical to those described in Section 2.2.2.

Player i's set of strategies is $S_i = \{(0,0),(0,1),(1,0),(1,1)\}$, where each pair $s_i = (n,t)$ denotes player i's strategy combination of n_i and t_i. In a two-player game, by investigating the possible outcomes, two equilibria are found:

1. mutual cooperation with both players having multiple partners, $s_1 = s_2 = (1,1)$
2. mutual defection with both players having multiple partners, $s_1 = s_2 = (1,0)$,

the latter being a prisoner's dilemma equilibrium. Nooteboom (1999) brings out the following hypotheses from the model:

- H5 Having multiple partners $(s_i = (1,t))$ dominates monogamy $(s_i = (0,t))$ if unconditional benefit is gained by multiplicity.
- H6 Monogamous mutual cooperation $(s_1 = s_2 = (0,1))$ is optimal if $c > m + v$. However, it is not an equilibrium.
- H7 Mutual defection $(s_1 = s_2 = (1,0))$ becomes an inefficient equilibrium if $c > p$.
- H8 Cooperation with multiple partners $(s_i = (1,1))$ is an equilibrium if $p > c$.

The situation of hypothesis H6 arises when the risk of spillover is greater than the benefits of having multiple partners and multiple sources of learning $(c > m+v)$. The hypothesis H7 comes into question if the risk of spillover is greater than the benefit of teamwork $(c > p)$, respectively H8 if $(p > c)$. Tacit knowledge often implies the case in H8; the risk of information spillover is low and successful joint production requires mutual teamwork.

Several *advantages* are gained as the number of partners increases:

1. sources of learning have more variation
2. risk is shared among a greater number of players
3. possibilities for bargaining grow better.

The following *disadvantages* are also the consequence of an increasing number of partners:

4. greater costs due to greater consumption of resources
5. risk of spillover increases.

Hence a player would like to have plenty of partners who all have few partners. However, since each of these partners would also like to be the "hub" of the network, Nooteboom (1999) formulates an optimisation problem, as follows. Consider the marginal utility of nth partner to player i. According to points 1 to 5 the marginal utility is

$$du_i(n_i) = y - b \cdot (n_i - 1) - an_j, \tag{6}$$

which is decreasing in the number of partners (n_i) and partners' partners (n_j denotes the number of partners that partner j has). The net effect of points 1 to 4 is modelled by $y - b \cdot (n_i - 1)$ and the effect of point 5 by $-an_j$. The utility to i of having n partners is the sum

$$u_i(n_i) = \sum_{k=0}^{n_i} du_i(k) = n_i(y - an_j) - \frac{bn_i^2}{2}, \tag{7}$$

which is a concave function of n_i. Hence, the maximum utility is obtained by setting the first derivative to zero, yielding

$$n_i^* = \frac{y - an_j}{b}, \tag{8}$$

which is the optimal number of partners for i.

In game theoretic terms, n_i^* in (8) is i's *reaction curve* according to n_j. With exchanged indices i and j, formula (8) is j's reaction curve. The Nash equilibrium of optimal number of partners can be found in the intersection of i's and j's reaction curves:

$$n_i^* = n_j^* = \frac{y}{a+b}. \tag{9}$$

Nooteboom (1999) summarises the analysis with the following hypothesis:

H9 Three factors affect the number of partners. First, the number of partners increases with the additive benefit of having more partners (y). Secondly, the number of partners decreases with a higher risk of spillover (a) and, third, decreases with a faster decline in marginal utility gained from new partners (b).

A high value of y is characterised by the vital importance of new technology, whereas high a implies transparency of technology and high b implies complexity and narrow specialisation of competence.

2.3 Game Theoretic Models of Demand Supply Chain Management

Bakos and Brynjolfsson (1993) have carried out a comprehensive study of the optimal number of suppliers for a buyer. In their paper, Bakos and Brynjolfsson construct a game theoretic model on the basis that reducing the number of suppliers results in increased incentives for the suppliers to foster immaterial advancements. Immaterial investments such as better quality, responsiveness, and innovativeness are difficult – if not impossible – to describe in contracts between supplier and buyer.

Bakos and Brynjolfsson conclude that if the buyer sets great store by noncontractible investments, it may be optimal to have a small number of suppliers, regardless of search and transaction costs. However, relying on few suppliers is not always optimal. If the immaterial gains are of lesser importance, then it is optimal to increase the number of suppliers until the marginal cost of searching equals the expected marginal utility of having several suppliers. Second, if the immaterial investments from the buyer's part are of great importance, then the buyer gains bargaining power with the increasing number of suppliers.

Globally, the optimal number of partners is found somewhere in between the two extreme cases, i.e. having few partners versus having several partners. Bakos and Brynjolfsson suggest that the global optimum is achieved as an average of the two extreme-case optima, weighted by the relative importance of immaterial investments versus coordination costs.

Corbett and DeCroix (2001) have studied supplier incentives in the situation where the client is supplied with *indirect materials*. Indirect materials such as e.g. paint or other chemicals are not directly related to the final product manufactured, but are necessarily used at some point of the manufacturing process. In consequence, the client wants to reduce the use of the indirect material, whereas the supplier's profits depend on increasing volume.

The paper of Corbett and DeCroix (2001) analyses several contracts that motivate the supplier to cooperate with the buyer, in order to reduce the consumption of the indirect material. Game theoretic analysis shows that such contracts can always increase the profit of the supply chain, but do not necessarily entail reduction in consumption. Corbett and DeCroix also conclude that, generally, it is not practicable to reduce consumption *and* increase profits concurrently.

2.4 Information Sharing in a Network

Very often, improving the global efficiency of an enterprise network requires viable *flow of information* between the partners of the network. However, sharing confidential information, even between partner companies, is normally considered unfavourable in firms. A few game theoretic approaches to the problem have been published, among others those of Wolters and Schuller (1997) and Li (2002), which we shall briefly review.

Wolters and Schuller (1997) have developed a dynamic game theory model in order to study how a supplier and a buyer can be encouraged to trust each other. In the model, both players repeatedly have the opportunity to behave opportunistically, which terminates the partnership but yields benefit for the opportunistic party. The utility to both players grows sustainably as the game continues without exits.

The conclusion of Wolters and Schuller (1997) is that, in the sense of fostering trust, it is beneficial for the buyer to have fewer suppliers who are encouraged to engage in R&D cooperation with the buyer. The suppliers are rewarded by longer contracts, which provide economical stability and incentives for value-adding innovations.

Li (2002) investigates vertical information sharing, considering, on one hand, the total benefit for the network and, on the other hand, the disadvantage from the "direct effect" and the "leakage effect". By the former, Li (2002) means the opportunistic behaviour that a participant involved in the information sharing may engage in. The latter derives from the advantage that a competitor may attain by observing the actions of the informed parties.

Li's (2002) paper introduces two examples, in which the willingness to share information is examined. The first example illustrates a case where both the direct effect and the leakage effect discourage downstream network companies from sharing their demand information upwards in the network. Surprisingly, in the second example, the leakage effect acts *on the behalf* of the retailers that pass their cost information upwards to the distributors. The benefit from the leakage effect may even outweigh the disadvantage of the direct effect, thereby making openness attractive to the network companies.

2.5 Utility Sharing in a Network

Thus far, we have discussed means of *improving the efficiency* of enterprise networks. As a result of an efficiency arrangement, the network attains a utility, which is shared among the network companies. In game theoretic terms, such situations can be characterised as n*-person bargaining games*. The concept of bargaining in games was first introduced by Nash (1950) and thereafter discussed by e.g. Shapley (1953), Kalai and Smorodinsky (1975), and Roth (1979). Shapley's (1953) result, the *Shapley value*, was the first one to reckon with coalitions, which is an essential part in n-person games. Gul (1989) proved that the Shapley value is an applicable solution concept also in non-cooperative bargaining.

Thomson (2003) extensively studies different mechanisms for sharing the liquidation value of a bankrupt firm among the creditors. The case is not common for an enterprise network, although the similarity to utility sharing can be seen. Another recent paper on utility sharing is that of Ginsburgh and Zang (2003). They examine a situation where a syndicate of service providers offer the customer a limited-time access pass to their services. The customer pays the syndicate a fixed amount, independent of the usage of the services. In their paper, Ginsburgh and Zang (2003) suggest that the mechanism to be used in sharing the profit from the passes among the members of the syndicate is the Shapley value.

2.6 Conclusions on Literature Review

Few game theoretic studies on enterprise networks have been carried out. Some of the topics, among others, have been

- "free riding" in joint R&D projects
- contracts in supplier-buyer networks
- information sharing in a network
- utility sharing among network companies.

Most of the earlier papers can only give superficial guidelines for network management. Concrete help for decision-makers may be difficult to perceive, despite the paper of Ginsburgh and Zang (2003), which gives specific rules for utility sharing in the case at issue.

In Section 2.7 we introduce a game theoretic model that brings a wider perspective to the management of enterprise networks. Namely, the model combines the *mechanisms for innovation incentives* and the *systematic method for sharing utility*, which is attained by innovations. In addition, the model *allows the use of threats*, resp. *coalition formation* between network companies. Based on the literature review, the next section discusses game theoretic modelling in general. The general framework is then applied to construct the innovation-incentive model in Section 2.7.

2.7 Modelling Enterprise Networks as a Game

This section provides general rules by which an industry representative can construct *game theoretic models* of networking cases. *A model* is an imitation of a real-world structure often referred to as *a system*, which, for some reason, cannot be analysed in practice. A good model simulates the system as elaborately as is possible, for all practical purposes. Hence, the level of detail in the model has to be proportioned to the objective of the model. A convenient model can be utilised to study theoretically contemporary networking issues in a systematic manner. More generally, an introduction to mathematical modelling can be found in e.g. Ljung and Glad (1994).

In order to construct a game theoretic model of a network case, several elements have to be identified. First, the *case has to be bounded* and the *objective of the analysis* has to be fixed. Within each subsequent step of the modelling process, it is vital to recall the original object and compare the ongoing work with the purpose of using the model. Otherwise, if the original objective is lost, some parts of the model may easily become too detailed and the model itself will not be of any use in practice.

The second thing is to define the *players of the game*. This is usually a straightforward task; the players are the parties who are somehow involved in the situation. Usually, in a network economy, the players are companies of an enterprise network or companies that are in a vendor-customer relationship, which also intuitively seems evident.

Third, the players' *sets of available strategies* are defined. More clearly, a player's set of available strategies consists of the player's feasible alternative actions in association with the case being investigated. The available strategies are case-specific and depend on the context of the situation. For instance, if the object of interest is a joint R&D project of an enterprise network, the available strategies for one network company could be for instance

1. contribute to the project

or

2. "free ride" in the project.

The preceding example with two available strategies is a highly abstracted model. A more concrete set of available strategies would be, e.g. any amount of money between 0 € and 10000 €, which the company decides to invest in the R&D project. The use of threats and incentives also enter the concept of strategy. However, overtly dominated strategy options are to be excluded. The properties of the strategy sets have great influence on the structure and dynamics of the game. First, the amount of information available to players in the act of decision-making may crucially affect the players' decisions and thus has to be taken into account in the model. Second, the order of the players' decision-making has to be identified so that the dynamics of the game will be correctly modelled.

The fourth task in the modelling process is to construct the players' *utility functions*. Generally, each player has a different utility function, which may contain expectation values. A utility function determines the amount of utility that a player obtains from each possible combination of strategies that the players of the game may decide to execute. The utility functions play a major role in game theory since, naturally, each player tries to choose his strategy in such a way that his own utility is maximised. Thereby, for each player, there exists at least one strategy combination that is optimal for the player. It is noteworthy that a player is only capable of choosing his own action. Nevertheless, a player may influence the decisions of other players by e.g. negotiations, threats and coalitions.

When the basic elements – the players, the strategy sets and the utility functions – of the game have been constructed, the model is usable, in theory. However, the model is not complete, and should not be used, before it is *validated*. The validation process is the final cross-check that the model is applicable to the purpose it was originally planned for. At minimum, the validation process includes the following tasks:

- Recall the model's original purpose and cross-check the applicability of the model.
- Verify the amount of information that the use of the model demands and ensure the availability of the information.
- Verify that the model is solvable in terms of mathematical complexity.

If the validation process addresses a mismatch between the original objective and the structure of the model, then the necessary corrections to the model have to be made.

Otherwise, if the model seems valid for the purpose it was intended for, then the model can be applied to the analysis of the original case. A common objective of game theoretic analysis is to *predict the players' behaviour*. This is normally being done with the assumption that the players are rational and thus want to maximise their own utility. However, since the players' objectives might be conflicting, some sort of compromise or *equilibrium* of the game will be the focus of the analysis. The equilibria in different types of games are normally found by some form of optimisation, which is the commonly used mathematical means in game theoretic analysis. Depending on the case being studied, one can solve for e.g. equilibrium points, the optimal combination of strategies, bargaining outcomes, identification of potential of opportunism, coalition formation, etc.

In order to exploit the model later on as well, it has to be *maintained* by updating it whenever the surrounding circumstances change. A network economy is a dynamic environment and, thus, changes in the number of players, in the players' strategy sets, and in the players' interests are usual.

In the following, we present a stepwise pattern for game theoretic modelling of cases concerning network economy. Figure 3 depicts the modelling process.

Step 1. Outline the problem. Clarify the objective of the analysis and bear it in mind throughout the whole modelling process.

Step 2. Identify the players of the game. Usually, the players of the game are the companies that are involved in the case.

Step 3. Identify the players' available strategies. Here, it is important to recognise the strategy alternatives that are *essential* to the networking case being studied.

Step 4. Identify the players' utility functions. The purpose of a player's utility function is to furnish values for any combination of strategies that the players of the game may execute.

Step 5. Validate the model. The validation process cross-checks the applicability of the model to the purpose that it was originally planned for. If, for some reason, the model is not suitable, then resume Step 2.

Step 6. Apply the model. The way of using the model depends on the objective of the analysis, which has been determined in Step 1.

Step 7. Maintain the model. For future use, the model has to be updated when the circumstances change.

In summary, the basic elements of a game theoretic model are 1) the players of the game, 2) the available strategies of the players and 3) the utility functions of the players. The basic elements are common to all games and thus have to be identified in order to perform a game theoretic analysis. The interrelationships between players, the information structure, and the dynamics of the game are determined case-specifically.

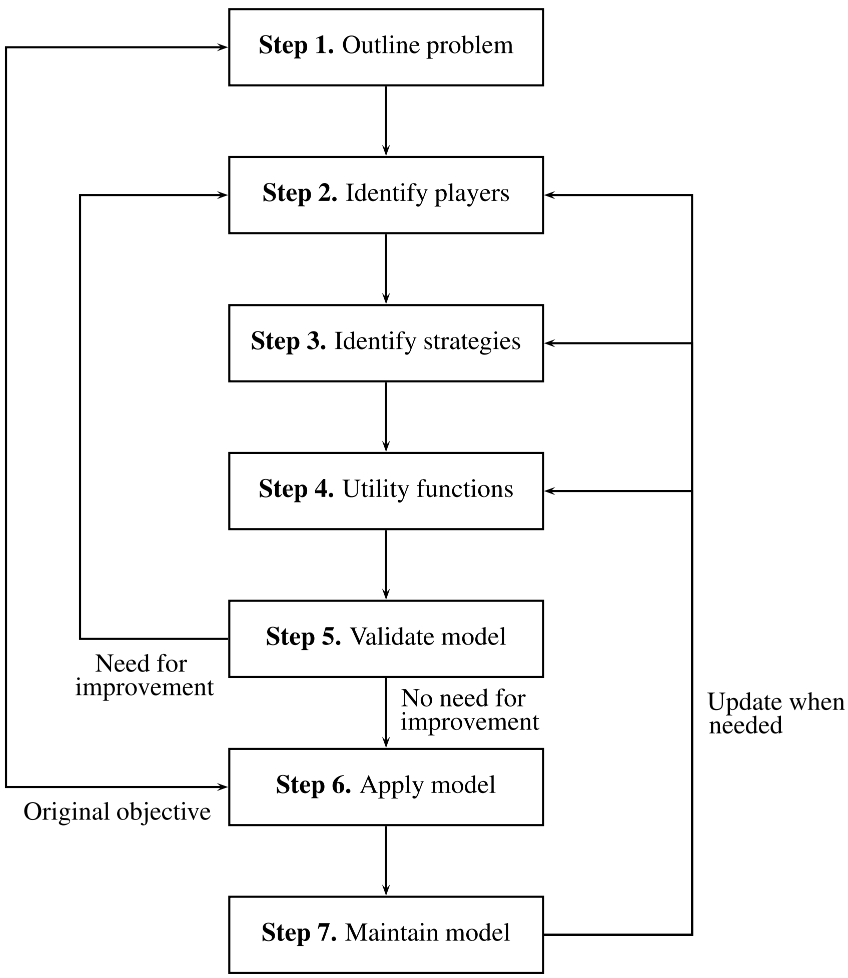

Figure 3. Game Theoretic Modelling Process.

3 Predetermined Innovation Incentives for Supplier Networks

3.1 Description of the Problem

Consider a network that consists of a *client* and several *suppliers* (Figure 4). The network forms a supply chain in which each of the companies carries out a specified task, which is part of the manufacturing of a final product. The client pays the suppliers a fixed payment p_i for their work w_i for each unit manufactured.

After having finished several products, it may happen that a possible rationalisation manoeuvre is identified in the manufacturing process. Our interest is in a situation where, in order to carry out the rationalisation, work has to be transferred from one supplier to another. Our aim is to *find a mechanism for redefining the payments* to the suppliers in such a situation. For instance, consider the following case:

Example 3.1 (Installing HPAC in boat building) *The client of this example is a Finnish sailing-yacht manufacturer* Nautor, *who has recently launched the production of a new type of boat, the* Swan 45. *Nautor has accumulated a network of suppliers, each working on one component of the new boat. Among others, there are two suppliers pertinent to this example: a hull manufacturer and a heating, plumbing and air-conditioning (HPAC) installer.*

In the construction of the very first boats, the HPAC installer himself drilled holes for the pipelines into the ready-made hull. Drilling the holes was time-consuming,

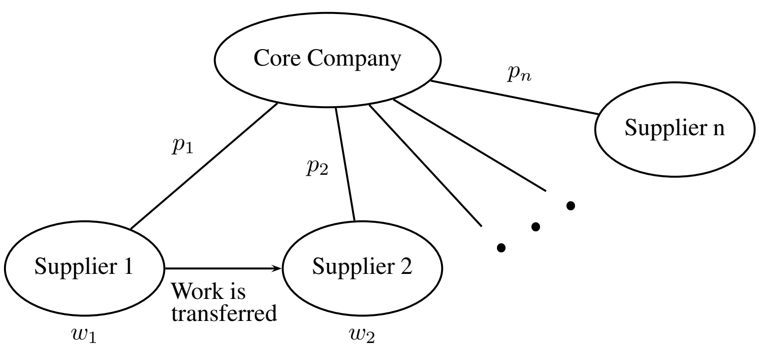

Figure 4. Enterprise Network with a Client and Several Suppliers.

since the drillman had to work in a constricted space and in uncomfortable positions. The striking change was to transfer the drilling of the holes from the HPAC installer to the hull construction, where the work could be done before the hull was assembled. Needless to say, this transfer of work speeded up the manufacturing process significantly.

The remaining question in Example 3.1 – and in numerous other cases beyond the scope of this study – is *how does the change in the suppliers' work load affect the prices paid by the client*? It seems apparent that the receiving supplier needs some compensation for the additional work in order to give his approval to the rationalisation. In addition, the mechanism should be an *incentive* to encourage the suppliers to develop such profitable solutions on their own initiative. Hence, a *predefined mechanism* for re-evaluation of the fixed payments is needed. In the following sections, we shall introduce a game theoretic approach to the problem and present a solution for the re-evaluation of payments.

3.2 Modelling the Work-Transfer Problem as Game

The players of the game are:

$1, \ldots, n-1$	the suppliers of the network
n	the client,

i.e. indices $1, \ldots, n-1$ denote the suppliers, and index n denotes the client. Let N denote the set of players $\{1, \ldots, n\}$ and let $N_s \subset N$ be $N_s = \{1, \ldots, n-1\}$.

In the *status quo*, a player $i \in N_s$ performs work w_i, for which he receives a positive payment p_i from n. Hence, *the profit* for player i is

$$\text{profit}_i = p_i - v_i, \tag{10}$$

where v_i is i's non-negative costs for the activity w_i (consisting of labour costs, material costs, etc.). Following the problem setting in Section 3.1, let Δw_i denote the change – negative or positive – in i's work load, and let $\Delta v_i(\Delta w_i)$ be the change in i's costs that depends on Δw_i. For convenience, $\Delta v_i(\Delta w_i)$ will be denoted by Δv_i in the following. It is reasonable to assume that Δv_i carries the same sign as Δw_i.

Let Δp_i denote the change in i's fixed payment due to the transfer of work. The *change in i's profit* is then

$$\pi_i(\Delta p_i) = \Delta p_i - \Delta v_i, \quad \forall\, i \in N_s. \tag{11}$$

Since player n makes the payments Δp_i to players $1, \ldots, n-1$, the change in his profit is

$$\pi_n(\mathbf{\Delta p}) = -\sum_{i=1}^{n-1} \Delta p_i, \tag{12}$$

where $\mathbf{\Delta p}$ denotes the vector $(\Delta p_1 \ \ldots \ \Delta p_{n-1})$. We shall use (11) as *the utility function for players $i \in N_s$* and (12) as *the utility function for player n*. Hence, in the *status quo* all the players' utilities are equal to zero, which is also the *disagreement outcome* of the game, as is illustrated in Figure 5. We assume that a player's utility increases linearly with respect to increase in profit. A generalisation to a non-linear (e.g. concave) utility function does not alter the implications of this study; it may complicate the calculus, though. Because the utilities are measured in monetary units, the assumption of *transferable utility* can be made (for the definitions, see e.g. Myerson 1997).

We present the problem as a two-stage game. In the first stage, the player n defines a rule ϕ, by which Δp_i's will be determined if the work load of players $1, \ldots, n-1$ change. Hence, the set of (pure) strategies available to player n is the family $\mathcal{F}$ of functions which map the changes in work load to changes in payments:

$$\phi \in \mathcal{F}, \quad \mathcal{F} = \{\phi \mid \phi : \mathbb{R}^{n-1} \mapsto \mathbb{R}^{n-1}\}. \tag{13}$$

Let ϕ_i denote the rule that concerns player i.

In the second stage, one or more of the players $1, \ldots, n-1$ discover(s) an option, which improves the efficiency of the network but requires transfer of work inside the network. Knowing the re-evaluation rule, he may now choose his strategy between coming up with the option $(c_i = a)$ or withholding the option $(c_i = b)$. Let us denote the set of strategies available to player $i \in N_s$ by $C_i = \{a, b\}$. The game in its *extensive form* is illustrated in Figure 5.

At the moment of decision-making, the players are familiar with the full history of the game, i.e. in Stage 2 player $i \in N_s$ knows the strategy ϕ that player n has chosen in Stage 1. Hence, the game is dynamic with *perfect information* (Luce and Raiffa 1957). In addition, since the utility function of each player is *common*

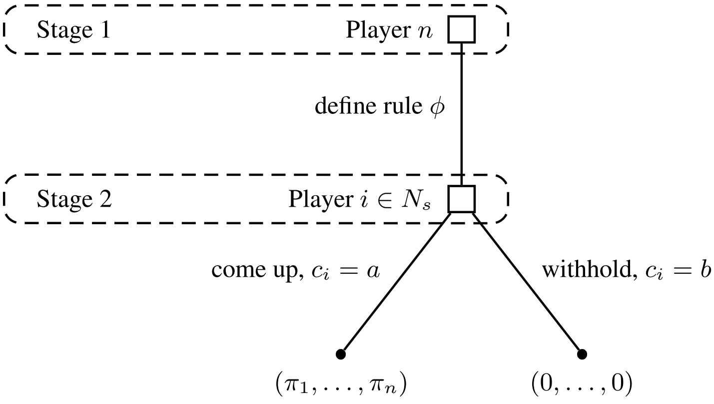

Figure 5. Game in the Extensive Form.

knowledge[2] among the players, the game is of *complete information* (Gibbons 1992).

Pulling the elements together, the work-transfer game in its *strategic form* is:

$$\Gamma = \left(N,\ (C_i)_{i \in N_s},\ \phi,\ (\pi_i)_{i \in N}\right), \tag{14}$$

where

N	is the set of players $N = \{1, \ldots, n\}$.	
N_s	is the set of suppliers $N_s = \{1, \ldots, n-1\}$.	
C_i	is the set of strategies available to players $i \in N_s,\ C_i = \{a, b\}$.	
ϕ	is a function $\phi : \mathbb{R}^{n-1} \mapsto \mathbb{R}^{n-1}$, which denotes the strategy of player n i.e. the rule defined by the client, $\phi = (\phi_1 \ldots \phi_{n-1})$.	(15)
π_i	denotes the utility to player $i \in N$. For $i \in N_s$, π_i is defined in (11) and for $i = n$, π_i is defined in (12).	

[2]For the definition of *common knowledge*, see Aumann (1976).

3.3 Analysis of Equilibria

In stage one, player n may act as a Stackelberg leader and choose his strategy in such a way that players $i \in N_s$ are encouraged to propose rationalising ideas (von Stackelberg 1934). It is therefore crucial to the outcome of the game that the player $i \in N_s$ has the information of the rule ϕ at the moment of decision-making (for Stackelberg games, see e.g. Başar and Olsder 1982, Gibbons 1992).

Assume that players $i \in N_s$ are in equivalent positions, in which case, without loss of generality, we can restrict our study to concern the utility of a single $i \in N_s$. A rational player $i \in N_s$ chooses to play $c_i = a$ if the consequences are profitable i.e. if

$$\pi_i > 0 \quad \overset{(11)}{\Longrightarrow} \quad \Delta p_i - \Delta v_i > 0. \tag{16}$$

We can replace Δp_i in (16) by $\phi_i(\Delta w_i)$, obtaining

$$\phi_i(\Delta w_i) > \Delta v_i, \tag{17}$$

which defines the *infimum* for $\phi \in \mathcal{F}$, on which player $i \in N_s$ is still willing to play $c_i = a$.

Again, replacing Δp_i in equation (12) by $\phi_i(\Delta w_i)$, the utility to player n becomes

$$\pi_n(\Delta w_i) = -\sum_{i=1}^{n-1} \phi_i(\Delta w_i). \tag{18}$$

Since the player n makes the final decision on the payments, it is necessary that his utility becomes positive:

$$\pi_n(\Delta w_i) > 0 \quad \Longrightarrow \quad \sum_{i=1}^{n-1} \phi_i(\Delta w_i) < 0. \tag{19}$$

Hence, we have obtained two criteria, namely (17) and (19), that enable the rationalisation manoeuvre to be put into practice. Replacing $\phi_i(\Delta w_i)$ in (19) by the right hand side of (17) gives us

$$\sum_{i=1}^{n-1} \Delta v_i < 0. \tag{20}$$

That is to say, the total change in the costs of the players' work load has to be negative, which also intuitively seems clear.

3.4 Definite Form for Incentive Strategy

We present one possible formulation for the rule ϕ_i. Let $N_w \subseteq N_s$ be $N_w = \{i \in N_s \mid \Delta w_i \neq 0\}$. Let $n_w = |N_w|$ be the cardinality of N_w. We construct ϕ on the following conditions:

$$\pi_n = \pi_i, \quad \forall \ i \in N_w \tag{21}$$

$$\pi_i = 0, \quad \forall \ i \in N_s \setminus N_w. \tag{22}$$

Condition (21) is to say that the players involved in the work transfer process $(i \in N_w)$ and the client (n) benefit equally. Condition (22) implies that the other players' payments stay unchanged. Placing (11) and (12) into (21) and replacing Δp_i's with ϕ_i's we obtain the following n_w equations:

$$-\sum_{i \in N_w} \phi_i = \phi_i - \Delta v_i, \quad \forall \ i \in N_w. \tag{23}$$

Equations (23) form a linear system of equations with n_w variables, which are the rules $\{\phi_i \mid i \in N_w\}$. Hence, solving the system for ϕ_i's leads to the unique solution (see Appendix A):

$$\begin{aligned}
\phi_i &= \frac{n_w}{n_w+1}\Delta v_i - \frac{1}{n_w+1} \sum_{\substack{j \in N_w \\ j \neq i}} \Delta v_j \\
&= \frac{(n_w+1)\Delta v_i - \Delta v_i - \sum_{\substack{j \in N_w \\ j \neq i}} \Delta v_j}{n_w+1} \\
&= \frac{(n_w+1)\Delta v_i - \sum_{j \in N_w} \Delta v_j}{n_w+1} \\
&= \Delta v_i - \frac{1}{n_w+1} \sum_{j \in N_w} \Delta v_j \qquad \forall \ i \in N_w,
\end{aligned} \tag{24}$$

giving each player an equal payoff of

$$\pi_i^* = -\frac{1}{n_w+1} \sum_{i \in N_w} \Delta v_i, \quad \forall \ i \in N_w \cup \{n\}. \tag{25}$$

In game theoretic terms, the allocation (24) is called an *egalitarian solution* since it satisfies both the *weak efficiency* and *equal-gains* conditions. Weak efficiency guarantees that all the utility will be shared among the players and equal-gains denotes that the players benefit equally. If the utilities in (21) are weighted with coefficients λ_i i.e.

$$\lambda_n \pi_n = \lambda_i \pi_i, \quad \forall \ i \in N_w, \quad \text{s.t.} \sum_{i \in N_w \cup \{n\}} \lambda_i = n_w + 1, \tag{26}$$

then the solution is called λ-*egalitarian*. Myerson (1997)

Furthermore, solution (24) satisfies the criteria (17) and (19) if Δv_i's satisfy condition (20). Note also that for each player $i \in N_w \cup \{n\}$ the maximum possible utility is

$$\begin{aligned} \pi_{\max} &= -\sum_{i \in N_w} \Delta v_i, \text{ when} \\ \pi_j &= 0, \quad \forall\, j \in N_w \cup \{n\},\ j \neq i. \end{aligned} \tag{27}$$

In addition, since allocation (24) satisfies the *greatest good* principle, it is said to be a *utilitarian solution*. An allocation that is both egalitarian and utilitarian is the *Nash bargaining solution* (see, Nash 1950, Myerson 1997, pp. 383).

Let F denote the set of *feasible divisions of utility* to π_i's, and let $\boldsymbol{\pi}$ denote the vector that consists of π_i's for $i \in N_w \cup \{n\}$. Since the game is *symmetric*, in the sense that (see e.g. Roth 1979)

$$\pi_{i,\min} = \pi_{j,\min} \quad \forall\, i,j \in N_w \cup \{n\} \quad \text{and} \tag{28}$$

$$\boldsymbol{\pi} \in F \quad \Longrightarrow \quad \text{every permutation of } \boldsymbol{\pi} \text{ is also contained in } F, \tag{29}$$

the egalitarian solution conforms to those presented by Nash (1950), resp. Kalai and Smorodinsky (1975). Usually, such bargaining solutions are applied only in two-player games because these solutions do not take account of *coalitions*, which may play an important role in n-player games (Luce and Raiffa 1957, pp. 155). Hence, a coalitional analysis of the game (14) will be studied in Section 3.7.

Figure 6 presents the graphical interpretation of the egalitarian solution in a three-player case (two suppliers and a client). The feasible region F forms a tetrahedron whose apex concurs with the origin $(\pi_i = 0\ \forall\, i \in N)$, and whose other vertices point the maximum utilities (27) for each player. The solution (25) is found in the tetrahedron's bottom-triangle centroid, i.e. the intersection of medians.

The implementation of the incentive strategy described in this section requires that *the suppliers* are familiar with *the change in their costs* (Δv_i) which arises due to the efficiency arrangement. Usually, companies have trouble-free access to such

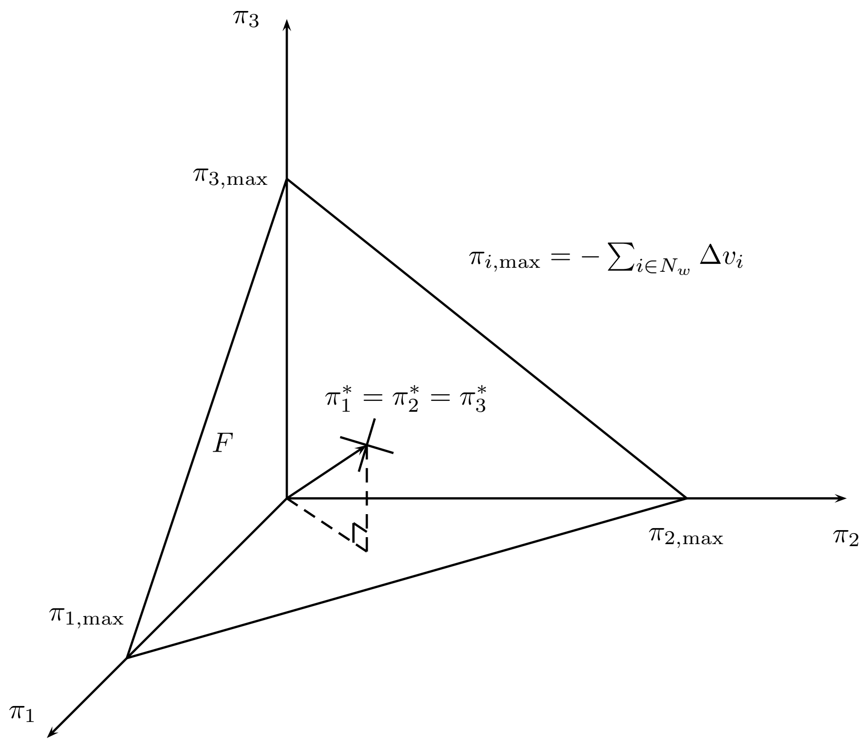

Figure 6. Feasible Region and the Utilities of Three Players.

basic information and hence the information requirement for players $1, \ldots, n-1$ is not an obstacle to the improvement of conditions. *The client* (player n), for one, needs this information from each player $1, \ldots, n-1$ individually. Whether this is possible depends on the internal relationships of the network, i.e. whether the suppliers trust their headman enough to share their confidential information with the headman. This case illustrates the importance of open-book management in enterprise networks.

3.5 Potential for Cheating

Cheating may occur in two forms. First, *the suppliers* may give untrue information about the change in their costs (Δv_i), trying to benefit more from the work-transfer operation. This lessens the share of utility to the other players and, in the extreme case, extinguishes the whole benefit from the operation. Therefore, it becomes less

<table>
<tr><td colspan="2"></td><td colspan="2">Player $j \in N_w$</td></tr>
<tr><td colspan="2"></td><td>truthful (t_j)</td><td>untruthful (u_j)</td></tr>
<tr><td rowspan="2">Player $i \in N_w$</td><td>truthful (t_i)</td><td>$(\pi_i^*,\ \pi_j^*,\ \pi_n^*)$</td><td>$(\pi_i^* - \beta_i,\ \pi_j^* + \beta_j,$
$\pi_n^* - \beta_n)$</td></tr>
<tr><td>untruthful (u_i)</td><td>$(\pi_i^* + \alpha_i,\ \pi_j^* - \alpha_j,$
$\pi_n^* - \alpha_n)$</td><td>$(\pi_i^* + \gamma_i,\ \pi_j^* + \gamma_j,$
$\pi_n^* - \gamma_n)$
or, if (32)
$(0,\ 0,\ 0)$</td></tr>
</table>

Figure 7. Cheating of Suppliers.

attractive for other players to accept the work transfer.

Consider the occasion of two suppliers, which is illustrated in Figure 7. Let π^*'s be the players' $1, \ldots, n$ payoffs that are obtained by the fair-play re-evaluation of the suppliers' payments (25). Let α's denote the changes in the players' utilities if i unilaterally distorts the truth to his own advantage $(c_i = u_i)$. In consequence, player i obtains the additive benefit $\alpha_i > 0$, whereas other players take losses maintaining the *zero sum*:

$$\alpha_i - \sum_{\substack{k \\ k \neq i}} \alpha_k = 0. \tag{30}$$

Respectively, let β's denote the shifts in utilities in the case of j's one-sided cheating, and let γ's denote the shifts if cheating is multilateral among the suppliers.

As can be seen in the Figure 7, telling the truth (t) is *strongly dominated* by being untruthful (u). Thus, the only Nash equilibrium of the game presented in Figure 7 is multilateral cheating (u_i, u_j). In such equilibrium, player n takes all the losses. However, since player n makes the final decision on the execution of the rationalisation manoeuvre, he might well decide not to put it into practice. To be specific, the rejection takes place if

$$\gamma_n > \pi_n^* \tag{31}$$

or, by equation (30),

$$\sum_{\substack{k \\ k \neq n}} \gamma_k > \pi_n^*. \tag{32}$$

In consequence, all the players gain zero utility, which turns the game into a *prisoner's dilemma*. Recognising this fact may reduce the willingness to cheat.

Another option for cheating appears to *the client*, who may decide not to follow the predefined rule for re-evaluation of the payments but, instead, compensate the suppliers less than what would be appropriate. However, such opportunistic behaviour would, obviously, lead to great mistrust between the suppliers and the client and hence to the rejection of all future cooperation.

Summarising this section, two possibilities for cheating are revealed. First, the suppliers might exaggerate or underrate their work loads or costs in order to obtain greater payments than they would fairly deserve. This, however, leads to a prisoner's dilemma where the utility to each player is zero. Hence, it does not pay for the suppliers to cheat. Secondly, the client may cheat by not following the predefined rule. Such an action is extremely hostile and, thus, can be ignored. As a conclusion, cheating in both cases is unprofitable.

3.6 Use of Threats

Originally, the concept of threats in negotiation was brought out by Nash (1953). By threat one means inconvenience – or lessening of utility – that a player can cause to another player. Nash's idea was that the players of the bargaining game each find an optimal threat, which is to be executed if the negotiations fail. That is, the threats define a new disagreement point for the bargaining game.

In the bargaining solution, a player's utility may increase as the other player's utility in the disagreement point decreases. This *chilling effect* makes the players want to set their threats so that the other player's state in the case of disagreement seems as unfavourable as possible. Formally, let (τ_1, τ_2) be the threats of players 1 and 2. Let $\omega_i(\tau_1, \tau_2)$ be the utility to player i in the bargaining solution with the disagreement point (τ_1, τ_2). Each player wants to set his threat so as to maximise his value of the game:

$$\tau_1^* = \arg\max_{\tau_1} \omega_1(\tau_1, \tau_2^*) \quad (33)$$

$$\tau_2^* = \arg\max_{\tau_2} \omega_2(\tau_1^*, \tau_2). \quad (34)$$

In his paper, Nash (1953) has proven the existence of the optimal threats (τ_1^*, τ_2^*) using the Kakutani fixed-point theorem (Kakutani 1941).

The game (14) described in Section 3.2, however, is not a pure bargaining game, but contains successive decisions in it. For dynamic games, Gibbons (1992, pp. 55) has introduced the notion *credibility of threats*. Defined complementarily, a threat is *noncredible* if the executor of the threat, in the situation of disagreement, is himself worse off when executing the threat than he would be by restraining himself from executing the threat. In other words, one should not believe a threat that is harmful also to the executor of the threat. Credibility of the threats is an essential part of the threat analysis, since noncredible threats can normally be ignored in the analysis.

In a supplier-buyer relationship a threat could be e.g. a termination of the contract. More generally, within enterprise networks, some examples of threats are (in the order of severity):

- segmentation/categorisation of the partner
- alteration in prices
- termination of partnership
- revelation of confidential information to e.g. a competitor
- hostile takeover of the opponent.

Of the above list, the last four points are self-explanatory, but the first point needs more explanation. Assume that company A is in a somewhat leading role in an enterprise network. The company A, therefore, has the option of categorising its partners into distinct classes, for instance, in order of lessening closeness: *strategic partners, R&D partners, business partners, etc.* (see e.g. Vesalainen 2002, pp. 40). Company A could set a threat of changing the classification of another company in the network to a lower class in A's grading.

Because in game (14) an alteration in prices p_i is exactly what we have already discussed in the previous sections, there is no point in using such a threat. Instead, it may be fruitful to examine *what happens if the suggested alteration in prices is not commonly accepted*. Thus, in analysing the threats in game (14), this study concentrates on the third item in the list above. Therefore, assume that each player $i \in N$ has an additional possible strategy, threat τ_i, which is the termination of the partnership. The threats $\tau_i \; \forall \; i \in N$ have to be added into the dynamic model of the game, depicted in the game tree of Figure 5. Because of the nature of the game, it is

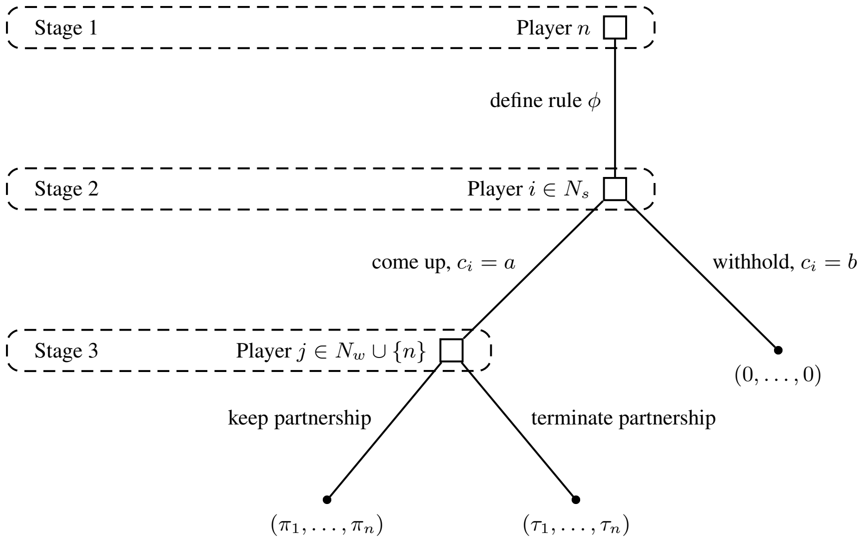

Figure 8. Game in the Extensive Form Including Threats.

not relevant to consider that the players would threaten each other, unless the game has advanced beyond Stage 2 and the player $i \in N_s$, who is making the decision in Stage 2, has decided to come up with his idea i.e. play $c_i = a$. Thus, Figure 8 expands the game tree to include the possibility of threatening to terminate the partnership.

In the situation of Figure 8, the threat τ_i is noncredible if and only if $\tau_i < 0$. That is, a rational player would not realise a threat that causes losses to himself, because, by maintaining the current conditions he would be better off. In consequence, if the companies of the network are mutually dependent on each other, there is no credible threat of terminating the partnership, and the threats can be ignored.

If, however, there exists a player j such that $\tau_j \geq 0$ and $\tau_n < 0$, then player j's threat is credible and should be taken into account. This is the case where supplier j has an optional client who is at least as profitable as player n, whereas for n, loosing supplier j would cause losses. These circumstances motivate player j to ask for more than π_j^*, which would be the payoff from the *egalitarian solution* $\boldsymbol{\pi}^*$ in (25).

One possible solution is to generalise Nash's (1953) theory to n players. Mathematically, this is straightforward. The Nash product for n players becomes

$$\prod_{i=1}^{n}(x_i - \delta_i), \tag{35}$$

where x_i is the share of utility to player i in cooperation, and δ_i is the disagreement payoff to player i. The maximisation of the Nash product (35) defines a unique strongly efficient vector $\mathbf{x}$, which is the Nash solution to the n-player bargaining problem (see Myerson 1997, pp. 417).

Hence, the share of utility in the threat game presented in Figure 8 can be defined by the unique strongly efficient vector $\boldsymbol{\pi} \in F$ that maximises the Nash product:

$$\boldsymbol{\pi}^* = \arg\max_{\boldsymbol{\pi}} \prod_{i \in N_w \cup \{n\}} (\pi_i - \tau_i). \tag{36}$$

The maximisation of (36) is equivalent to solving the following conditions

$$\pi_i - \tau_i \;=\; \pi_j - \tau_j \qquad\qquad \forall\;\; i,j \in N_w \cup \{n\} \tag{37}$$

$$\sum_{i \in N_w \cup \{n\}} \pi_i \overset{(27)}{=} -\sum_{i \in N_w} \Delta v_i. \tag{38}$$

Conditions (37) and (38) form a linear system of $|N_w|+1$ equations containing the same number of unknown variables (the π_i's). Thus, solving the system for π_i's defines vector $\boldsymbol{\pi}$ uniquely (see Appendix B).The compensation payment ϕ can then be calculated from (11):

$$\phi_i(\boldsymbol{\tau}) = \pi_i(\boldsymbol{\tau}) + \Delta v_i, \tag{39}$$

where $\boldsymbol{\tau}$ denotes the vector that consists of τ_i's such that $i \in N_w \cup \{n\}$.

3.7 Coalitional Analysis

3.7.1 Motivation and Basic Concepts

In Section 3.3, the game has been generalised to n players without considering *coalitions*. However, as the following example illustrates, there exists a need for coalitional analysis:

Example 3.2 (Installing HPAC in boat building) *Recall the Example 3.1 of Section 3.1. There are three players in this game, namely, the HPAC installer* $\{1\}$*, the* *hull manufacturer* $\{2\}$*, and the client* $\{3\}$*. Let us assume that, in the occasion of work transfer, player 3 has proactively decided to apply the egalitarian solution (24) to re-evaluate the payments to players 1 and 2, and that this is common knowledge.*

Suppose that player 1 has an idea of transferring the drilling work to player 2. Player 1 now has to decide, whether he will bring the possibility of the rationalisation manoeuvre to common knowledge. As a rational player, he will make this decision according to his expectations on the share of the total utility attained by the work transfer (Figure 5). Evidently, a coalition of players 1 and 2 could be better off leaving the player 3 uninformed of the work transfer and, thereby, sharing the total utility (27) bilaterally. That is, since player 3 does not bring any added value to the game, neither should he gain from the game "for free".

However, if player 3 requires that he is familiar with the manufacturing process, e.g. to assure certain quality in the final product, he may accuse players 1 and 2 of breaking the contract when the trickery comes to light (the consequences of which depend on the contract, etc.).

The Example 3.2 motivates the following coalitional analysis. First, the concepts of coalitions in game theory are briefly reviewed (for a detailed description, see e.g. Myerson 1997). A coalition or *syndicate* is any nonempty subset S of N:

$$S \subseteq N, \quad S \neq \emptyset. \tag{40}$$

The *characteristic function* $\nu(S)$ is a function

$$\nu : \mathcal{P}(N) \mapsto \mathbb{R}, \tag{41}$$

where $\mathcal{P}(N)$ is the *power set* of N. The characteristic function represents the amount of utility that a coalition $S \subset N$ can guarantee to its members, regardless of the strategies of the players in the competing coalition $N \setminus S$ (see e.g. Jones 1980, Myerson 1997). Occasionally, $\nu(S)$ is referred to as the *worth* of coalition S.

In the literature, several different definitions of characteristic functions have been presented, some of which will be briefly discussed here. The first method to define the characteristic function is a *minimax* representation, suggested by von Neumann and Morgenstern (1944). For a coalition $S \in N$, the characteristic function is

$$\nu(S) = \min_{c_{N\setminus S}} \max_{c_S} \sum_{i\in S} \pi_i(c_S, c_{N\setminus S}), \tag{42}$$

where c_X denotes the strategies of players in coalition X. In other words, the worth of coalition S is the sum of individual utilities to the members of S when the complementary coalition $N \setminus S$ plays its most offensive strategy against S. It can be shown that the characteristic function ν in (42) satisfies two properties, stated by Luce and Raiffa (1957):

$$\begin{array}{ll}\text{(i)} & \nu(\emptyset) = 0 \\ \text{(ii)} & S, T \subset N,\ S \cap T = \emptyset \implies \nu(S \cup T) \geq \nu(S) + \nu(T).\end{array} \tag{43}$$

For the proof, consult e.g. Jones (1980, pp. 187). The first property (i) is to say that a coalition with no players neither achieves nor loses anything. The second property (ii) is called *superadditivity*, that is a union of two coalitions can perform at least as well as the two separate coalitions. The property (ii) is called *additivity* if, instead of the inequality, only the equality holds, i.e. if (Jones 1980)

$$\nu(S \cup T) = \nu(S) + \nu(T) \quad \forall\, S, T \subset N,\ S \cap T = \emptyset. \tag{44}$$

Further, a coalitional game is called *inessential* if its characteristic function is additive. Other coalitional games are called *essential* (Luce and Raiffa 1957). Inessential games are not interesting in the sense of coalitions, since each player is able to guarantee himself a certain amount of utility, regardless of alliances.

Another way of defining the characteristic function is the payoff when both coalitions S and $N \setminus S$ play their defensive strategies

$$c_S^* = \arg\max_{c_S} \sum_{i \in S} \pi_i(c_S, c_{N\setminus S}^*) \tag{45}$$

$$c_{N\setminus S}^* = \arg\max_{c_{N\setminus S}} \sum_{i \in N\setminus S} \pi_i(c_S^*, c_{N\setminus S}). \tag{46}$$

That is, both coalitions maximise their own utility assuming that the other coalition behaves similarly. The strategy pair (45) and (46) is called the *defensive equilibrium representation* (Myerson 1997). The problem with this definition is that the pair $(c_S^*, c_{N\setminus S}^*)$ does not necessarily exist. Nonetheless, if such a strategy pair exists, then the characteristic function is defined to be

$$\nu(S) = \sum_{i \in S} \pi_i(c_S^*, c_{N\setminus S}^*). \tag{47}$$

It is noteworthy that the defensive equilibrium (45)-(46) is a sort of generalisation of the Nash equilibrium of non-cooperative games (see, Nash 1951).

Harsanyi (1963) proposes yet another definition for the characteristic function, which is a generalisation of Nash's (1953) *rational threats* criteria to the n-player game. Harsanyi's idea is that, instead of maximising merely the total utility (45), a coalition should maximise the *difference between its own total utility and the competitors' total utility*. Thus, the coalitions' optimal strategies become

$$c_S^* = \arg\max_{c_S} \left(\sum_{i \in S} \pi_i(c_S, c_{N \setminus S}^*) - \sum_{j \in N \setminus S} \pi_j(c_S, c_{N \setminus S}^*) \right) \tag{48}$$

$$c_{N \setminus S}^* = \arg\min_{c_{N \setminus S}} \left(\sum_{i \in S} \pi_i(c_S^*, c_{N \setminus S}) - \sum_{j \in N \setminus S} \pi_j(c_S^*, c_{N \setminus S}) \right). \tag{49}$$

Again, the characteristic function is calculated from (47), this time with the strategies $(c_S^*, c_{N \setminus S}^*)$ obtained from (48) and (49).

3.7.2 Core and Coalition Formation in the Work-Transfer Game

Bearing in mind the original goal, which is to determine feasible incentives for the suppliers, we must find a means to predict the formation of coalitions in the supplier-client game. Gillies (1953) has introduced the concept *core of a coalitional game*, which we define as follows. Let vector $\mathbf{x} = (x_1, \ldots, x_n)$ be any payoff allocation, or *imputation*, to players $i \in N$. Imputation $\mathbf{x}$ is in the *core* of a coalitional game, if

$$\sum_{i \in N} x_i = \nu(N) \quad \text{and} \quad \sum_{i \in S} x_i \geq \nu(S) \quad \forall \ S \subseteq N. \tag{50}$$

That is to say, an imputation $\mathbf{x}$ is in the core, if no coalition can insure its members a strictly better payoff than in $\mathbf{x}$. Hence, a coalition can be assumed stable if and only if it is able to offer its members a utility allocation that is in the core. Within the definition of the core (50), it is natural to use the minimax representation (42) of the characteristic function, because then players in S can assure themselves a strictly better payoff than in $\mathbf{x}$, regardless of what the players in $N \setminus S$ do, if and only if $\sum_{i \in S} x_i < \nu(S)$ (Luce and Raiffa 1957, Jones 1980, Myerson 1997).

In the game of Example 3.2, assuming that there are no threats of any kind and that the characteristic function is (42), the core is

$$\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 + x_2 = -(\Delta v_1 + \Delta v_2), \ x_3 = 0\}. \tag{51}$$

That is, the suppliers 1 and 2 share the utility obtained from the rationalisation manoeuvre, and player 3 is left with zero payoff.

3.7.3 Threats in Coalitional Analysis of the Work-Transfer Game

As presented in Section 3.6, the work-transfer game (14) includes the possibility of two kinds of threats. Firstly, a supplier possesses the threat of terminating the contract, similar to the case in the analysis of Section 3.6. Secondly, the client can also threaten the suppliers with termination of the contract, if two or more of the suppliers ally against the client, and the client gains the information that the coalition has been formed. Since each supplier is in an equivalent position, we present this threat game as a two-player game between a supplier and the client (see Figure 9). For each $i \in N_w, j \in N$, let $\tau_j(i)$ denote the utility to player j if the contract between i and n is terminated. For convenience, we write $\tau_i(i) = \tau_i, \; \forall\, i \in N_w$.

Of the outcomes of the game presented in Figure 9, let us make the following assumptions:

$$\pi_n > 0 \tag{52}$$

$$x_i > \pi_i > 0. \tag{53}$$

Thus, independent of the values of τ's, for player $i \in N_s$ both strategies a and c weakly dominate the strategy b of withholding the rationalisation idea (see Fig-

Player n

Player $i \in N_w$	cooperate (α)	prevent rationalisation (β)	terminate contract (γ)
cooperate (a)	$(\pi_i,\ \pi_n)$	$(0,\ 0)$	$(\tau_i,\ \tau_n)$
withhold (b)	$(0,\ 0)$	$(0,\ 0)$	$(\tau_i,\ \tau_n)$
ally against client (c)	$(x_i,\ 0)$	$(0,\ 0)$	$(\tau_i,\ \tau_n)$
terminate contract (d)	$(\tau_i,\ \tau_n)$	$(\tau_i,\ \tau_n)$	$(\tau_i,\ \tau_n)$

Figure 9. Coalition Formation Between Suppliers with Threats of Terminating Contract.

		Player n	
		cooperate (α)	terminate contract (γ)
Player $i \in N_w$	cooperate (a)	$(\pi_i,\ \pi_n)$	$(\tau_i,\ \tau_n)$
	ally against client (c)	$(x_i,\ 0)$	$(\tau_i,\ \tau_n)$
	terminate contract (d)	$(\tau_i,\ \tau_n)$	$(\tau_i,\ \tau_n)$

Figure 10. Coalition Formation Between Suppliers with Threats of Terminating Contract, Reduced Game.

ure 8), which would provide the zero payoff to all players (unless termination of contract by player n). Again, independent of the values of τ's, for player n playing β is weakly dominated by playing α. Hence, the game is substantially equivalent to that in Figure 10, in which the dominated strategies have been omitted.

In order to make the threat analysis interesting, let us assume

$$\begin{aligned} &\tau_i < 0 \text{ and } 0 \leq \tau_n < \pi_n && \text{(a)} \\ \text{or}& \\ &0 \leq \tau_i < \pi_i \overset{(53)}{<} x_i \text{ and } \tau_n < 0. && \text{(b)} \end{aligned} \tag{54}$$

Otherwise, if both τ's were positive, then the contract would already have been proactively terminated. On the other hand, if both τ's were negative, then the threats would be noncredible and no player would terminate the contract, having the alternative of keeping the current contract. Thus, with the assumption (54) there is always one player who possesses a credible threat against another. Furthermore, no player gains so much from terminating the contract that termination would dominate all other strategies without question.

In the instance of (54a), the unique non-cooperative Nash equilibrium is (c, γ), which is the only strategy pair that survives the iterated elimination of dominated strategies. Elaborately, considering Figure 10, c dominates a and d, in which case player n executes his credible threat γ. On the contrary, given (54b), the unique Nash equilibrium of the game is (c, α).

Henceforth, we shall consider the situation as a whole, i.e. between not only one supplier and the client, but with all the players $i \in N_w$ included. Let us divide the set N_w into two disjoint subsets, namely W_a and W_b, such that

$$W_a = \{i \in N_w \mid 0 \leq \tau_n(i) < \pi_n\} \tag{55}$$
$$W_b = \{i \in N_w \mid \tau_n(i) < 0\}. \tag{56}$$

In other words, players in W_a are more dependent on the client n than players in W_b. This is due to the credible threat that player n possesses against players in W_a. Against players of W_b, player n has no credible threats. Assume further, that $W_a \cap W_b = \emptyset$ (by definition) and that $W_a \cup W_b = N_w$, which is a direct implication of the assumption (54a).

Let us define the *coalitional threat game* as a generalisation of the game (14) as follows:

$$\nu = \left(S \subseteq (W_a \cup W_b \cup \{n\}),\ C_S,\ \nu(S)\right), \tag{57}$$

where $C_S = \times_{i \in S} C_i$ is the set of strategies of the players in S and $\nu(S)$ is a characteristic function.

Evidently, if $W_a = \emptyset$, then the core of the coalitional game (57) is

$$\{\mathbf{x} \in \mathbb{R}^n \mid \sum_{i \in N_w} x_i = -\sum_{i \in N_w} \Delta v_i,\ \sum_{j \in N \setminus N_w} x_j = 0\}. \tag{58}$$

That is, players $i \in N_w$ share the whole utility gained within the rationalisation manoeuvre, leaving the client n with zero utility. This is possible, because n's threats against the players in N_w are not credible.

However, if $\exists\, i \in W_a$, then player n possesses a credible threat against i and, since $N_w \setminus \{i\}$ are not able to perform the rationalisation manoeuvre without i, the disagreement payoff for each $j \in N_w$ is $\tau_j(i)$, which is assumed to be inefficient (54). Therefore, if $W_a \neq \emptyset$, the core is

$$\{\mathbf{x} \in \mathbb{R}^n \mid \sum_{i \in N_w \cup \{n\}} x_i = -\sum_{i \in N_w} \Delta v_i,\ \sum_{\substack{j \in N \\ j \notin N_w \cup \{n\}}} x_j = 0\}. \tag{59}$$

Let us call $S_g = N_w \cup \{n\}$ the *grand coalition*. The results (58) and (59) seem intuitively clear; unless the client is in a strong – such as monopsonistic, etc. –

position against at least one of the suppliers, the network of suppliers can form coalitions wherein e.g. work can be transferred and the utility derived therefrom can be shared inside the coalition. On the other hand, if the suppliers are dependent on the client, then he is able to disrupt the coalition of suppliers, and no coalition except for the grand coalition will manage to succeed. Within the grand coalition, the client is also able to claim a share of the total utility.

3.7.4 Shapley Value

The core as a solution concept has several limitations. First, the core can be empty. In fact, Jones (1980, pp. 202) has proven that the core of any essential constant sum game is empty. Second, the core can be very sensitive to small changes in the power structure of the game (see e.g. Myerson 1997). Third, the core of an essential game, when nonempty, does not uniquely define the appropriate imputation (Luce and Raiffa 1957, Jones 1980).

A more elaborate means of finding an outcome for n-player bargaining is the *Shapley value*, which was introduced by Shapley (1953). In his paper, Shapley composes three axioms that the bargaining outcome should satisfy, and he then proves that such allocation exists and that the allocation is unique. The Shapley value for player i of a coalitional game ν is

$$\varphi_i(\nu) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} \left(\nu(S \cup \{i\}) - \nu(S)\right), \qquad (60)$$

where $\nu(X)$ is the characteristic function (the worth) of coalition X.

An intuitive interpretation of the Shapley value is as follows. Assume that the $|N|$ players of the game are outside of an empty room and that there exists one door that leads into the room. Then there are $|N|!$ possible ways how the players can line up at the door. Assume that the players line up randomly in the queue and that each ordering is equally likely. For player i, there are $|S|!$ different ways to order a coalition S *in front of* player i in the queue. Respectively, there are $(|N|-|S|-1)!$ ways to order the rest of the players *behind* i. Hence, if the players enter the room one at a time in the order determined by the queue, then the probability that i, when he enters the room, finds the coalition S there is $\frac{|S|!(|N|-|S|-1)!}{|N|!}$. Player i's additional value to coalition S is $\nu(S \cup \{i\}) - \nu(S)$. Thus, i's Shapley value is his *expected additional value to a coalition that has formed into the room ahead of him*.

In calculation of the Shapley value of game (14), if no threat strategies are existent, we can use the characteristic function of (42), suggested by von Neumann and Morgenstern (1944). In this case, the Shapley values for the players of the game presented in Section 3.2 become

$$\pi_i = \varphi_i = \begin{cases} \frac{1}{|N_w|}\pi_{\max} & \forall\, i \in N_w \\ 0 & \forall\, i \in N \setminus N_w, \end{cases} \tag{61}$$

where $\pi_{\max}$ is the total utility to be shared, defined in (27). In other words, the suppliers that are involved in the work-transfer process share the whole utility of the rationalisation manoeuvre. By symmetry, each supplier $i \in N_w$ gains the same amount of utility. Note also that the core in (51) contains the allocation (61).

3.7.5 Harsanyi's Modified Shapley Value

Harsanyi (1963) has extended the Shapley value to comprise the use of threats, as well as the case of *non-transferable utility*. The NTU games are left beyond the scope of this study. However, as has been discussed already, the possibility of using threats is an essential part of the game. For this purpose, Harsanyi (1963) presents a *modified Shapley value*, which is calculated from the original formula (60) but with a particular characteristic function $\nu(S)$. Contrary to the von Neumann and Morgenstern (1944) characteristic function, $\nu(S)$ in the modified Shapley value is to be determined from (47) using the strategies obtained from equations (48) and (49).

The rest of this section presents the solution to the work-transfer game according to the modified Shapley value. Recall the division of set N_w into two disjoint subsets W_a (55) and W_b (56). We again restrict the analysis to two interesting cases, namely

1. $W_a = \emptyset$
2. $W_a \neq \emptyset$.

The first case implies that player n possesses no credible threats against any of the players in N_w. Hence, the modified Shapley values are equal to those presented in (61). The latter case, however, entails a different solution, the modified Shapley values of which will be calculated first for players in W_a, then for players in W_b and finally for player n, as follows.

Let $\bar{S}$ denote the set of players that are left out of the coalition $S \cup \{i\}$ i.e. $\bar{S} = (N_w \cup \{n\}) \setminus (S \cup \{i\})$. The marginal utility of a player $i \in W_a$ to coalition S is

$$\nu(S \cup \{i\}) - \nu(S) = \begin{cases} \pi_{\max} - \tau_n(i), & \bar{S} = \emptyset \\ -\tau_n(i), & n \in S,\ \bar{S} \neq \emptyset \\ \tau_i, & n \notin S. \end{cases} \tag{62}$$

That is, if i is the last player to enter the grand coalition $S_g = N_w \cup \{n\}$, then i is the critical player who allows the rationalisation manoeuvre to be put into practice. Therefore, since it is necessary for the execution of the rationalisation manoeuvre that the grand coalition $N_w \cup \{n\}$ is formed, the last player to join the grand coalition carries the total utility $\pi_{\max}$.

If i and n are part of a same coalition, then it does not pay for n to execute his threat of terminating the contract with i. Hence, when i enters a coalition S that n belongs to, the coalition loses the value of n's threat i.e. $\tau_n(i)$ (≥ 0 by 54a and 55). On the other hand, if i enters a coalition S, such that $n \notin S$, then i carries the cost of n's threat i.e. τ_i (< 0 by 54a and 55).

The probability that i is the last player to join the grand coalition is

$$P(\bar{S} = \emptyset) = \frac{1}{|N_w| + 1}. \tag{63}$$

The probability that when i enters S, n belongs to S and $S \cup \{i\}$ does not form the grand coalition, is

$$\begin{aligned} P(n \in S,\ \bar{S} \neq \emptyset) &= \sum_{k=1}^{|N_w|-1} \frac{k}{|N_w|} \cdot \frac{1}{|N_w| + 1} \\ &= \frac{1}{|N_w| \cdot (|N_w| + 1)} \sum_{k=1}^{|N_w|-1} k \\ &= \frac{1}{|N_w| \cdot (|N_w| + 1)} \cdot \frac{1 + |N_w| - 1}{2} \cdot (|N_w| - 1) \\ &= \frac{1}{2} \cdot \frac{|N_w| - 1}{|N_w| + 1}. \end{aligned} \tag{64}$$

The probability that n is not a member of S is

$$P(n \notin S) = \sum_{k=1}^{|N_w|} \frac{k}{|N_w|} \cdot \frac{1}{|N_w| + 1}$$

$$
\begin{aligned}
&= \frac{1}{|N_w| \cdot (|N_w|+1)} \cdot \frac{1}{2} \cdot |N_w| \cdot (|N_w|+1) \\
&= \frac{1}{2}. \qquad (65)
\end{aligned}
$$

Thus, combining the marginal utilities (62) and the corresponding probabilities (63)-(65), the expected marginal utility of $i \in W_a$ to coalition S becomes

$$
\begin{aligned}
\varphi_i &= \frac{\pi_{\max} - \tau_n(i)}{|N_w|+1} - \frac{\tau_n(i)}{2} \cdot \frac{|N_w|-1}{|N_w|+1} + \frac{\tau_i}{2} \\
&= \frac{\pi_{\max}}{|N_w|+1} - \frac{\tau_n(i)}{2} + \frac{\tau_i}{2}, \qquad i \in W_a, \qquad (66)
\end{aligned}
$$

which is the modified Shapley value for a player $i \in W_a$.

For a player $j \in W_b$ the situation is simpler. The marginal utility of a player $j \in W_b$ to coalition S is

$$
\nu(S \cup \{j\}) - \nu(S) = \begin{cases} \pi_{\max}, & \bar{S} = \emptyset \\ 0, & \bar{S} \neq \emptyset. \end{cases} \qquad (67)
$$

Again, if j is the last player to enter the grand coalition $S_g = N_w \cup \{n\}$, then j carries the total utility $\pi_{\max}$. Otherwise, if $S \cup \{j\}$ does not form the grand coalition, then j's additional value to S is zero. The probability $P(\bar{S} = \emptyset)$ is equal to that in (63). Thus, the modified Shapley value for a player $j \in W_b$ becomes

$$
\varphi_j = \frac{\pi_{\max}}{|N_w|+1}, \quad j \in W_b. \qquad (68)
$$

Finally, let us determine the modified Shapley value for player n. The marginal utilities are

$$
\nu(S \cup \{n\}) - \nu(S) = \begin{cases} \pi_{\max} - \sum\limits_{i \in W_a} \tau_i, & \bar{S} = \emptyset \\ \sum\limits_{i \in W_a \cap \bar{S}} \tau_n(i) - \sum\limits_{i \in W_a \cap S} \tau_i, & \bar{S} \neq \emptyset. \end{cases} \qquad (69)
$$

In other words, if n completes the grand coalition, he carries the total utility of the rationalisation manoeuvre and, in addition, removes the burden of the threat of terminating contracts between n and the players in W_a. Again, if $\bar{S}$ is left unempty, then the threats against players in $W_a \cap \bar{S}$ are enforced and the threats against players in $W_a \cap S$ are cancelled.

The probability $P(S = \emptyset)$ is once again obtained from (63). The conditional probabilities that a player $i \in W_a$ belongs to $\bar{S}$ when $|\bar{S}| = k$, or to S when $|S| = |N_w| - k$ are, respectively,

$$P(i \in \bar{S} \mid k) = \frac{k}{|N_w|} \tag{70}$$

$$P(i \in S \mid k) = \frac{|N_w| - k}{|N_w|}. \tag{71}$$

Hence, n's expected marginal utility to a coalition S is

$$\begin{aligned}
\varphi_n &= \frac{\pi_{\max} - \sum_{i \in W_a} \tau_i}{|N_w| + 1} \\
&\quad + \sum_{k=1}^{|N_w|} \frac{1}{|N_w| + 1} \left[\sum_{i \in W_a} \left(\frac{k}{|N_w|} \cdot \tau_n(i) - \frac{|N_w| - k}{|N_w|} \cdot \tau_i \right) \right] \\
&\ \ \vdots \qquad \text{(for the calculations, see Appendix C)} \\
&= \frac{\pi_{\max}}{|N_w| + 1} + \frac{\sum_{i \in W_a} \tau_n(i)}{2} - \frac{\sum_{i \in W_a} \tau_i}{2}.
\end{aligned} \tag{72}$$

Obviously, the modified Shapley value for the suppliers that are not involved in the rationalisation process is zero:

$$\varphi_k = 0, \quad k \in N_s \setminus N_w. \tag{73}$$

Summarising, the modified Shapley values of the coalitional threat game (57) for the players in N are

$$\begin{aligned}
\varphi_i &= \frac{\pi_{\max}}{|N_w| + 1} - \frac{\tau_n(i)}{2} + \frac{\tau_i}{2}, && i \in W_a && \text{(a)} \\
\varphi_j &= \frac{\pi_{\max}}{|N_w| + 1}, && j \in W_b && \text{(b)} \\
\varphi_k &= 0, && k \in N_s \setminus N_w. && \text{(c)} \\
\varphi_n &= \frac{\pi_{\max}}{|N_w| + 1} + \frac{\sum_{i \in W_a} \tau_n(i)}{2} - \frac{\sum_{i \in W_a} \tau_i}{2}. && && \text{(d)}
\end{aligned} \tag{74}$$

It is straightforward to verify that the players' modified Shapley values (74) sum up to the total available utility:

$$\sum_{i \in W_a} \varphi_i + \sum_{j \in W_b} \varphi_j + \varphi_n = \pi_{\max}, \tag{75}$$

that is, the allocation $\mathbf{\Phi} = (\varphi_1 \ \ldots \ \varphi_n)$ is efficient.

Since the players of W_a are in a weaker bargaining position than the players of W_b, the modified Shapley value for the players in W_a (74a) is strictly less than that for players in W_b (74b). In fact, players in W_b obtain the same amount of utility that the egalitarian solution (25) would give, and players in W_a forfeit an amount of utility – proportional to the strength of n's threat $(\tau_n(i),\ \tau_i)$ – to player n.

The incentive strategies ϕ_i and ϕ_j for $i \in W_a$ and $j \in W_b$ according to the modified Shapley values are obtained by replacing π_i in (11) by φ_i and φ_j, respectively:

$$\begin{aligned} \phi_i &= \Delta v_i + \frac{\pi_{\max}}{|N_w|+1} - \frac{\tau_n(i)}{2} + \frac{\tau_i}{2}, \quad \forall\, i \in W_a. && \text{(a)} \\ \phi_j &= \Delta v_j + \frac{\pi_{\max}}{|N_w|+1}, \quad \forall\, j \in W_b && \text{(b)} \end{aligned} \tag{76}$$

The incentives (76a) and (76b) take account of the fact that the suppliers in W_a are more dependent on the client n than the suppliers in W_b. Hence, it is reasonable that the incentive for the suppliers in W_a is lower than that for the suppliers in W_b.

3.8 Summary of Innovation Incentives Model

In this section, we have introduced a game that models efficiency-improving arrangements in enterprise networks. We have studied the conditions that a rationalisation manoeuvre requires in order to take place. Often, when a rationalisation manoeuvre is implemented, the costs of some network companies increase, whereas the costs of some other companies in the network reduce. The first criterion for the rationalisation is that the *total reduction in costs is greater than the total increase in costs*. The other criterion is that each of the network companies are better off after the rationalisation has been carried out. That is, the *companies whose costs increase need a compensation payment* in order to accept the implementation of the rationalisation manoeuvre. The main idea of our model is that when the network companies know that they will be compensated in the case of a rationalisation manoeuvre, then the companies are willing to implement efficiency-improving ideas. Hence, the model determines appropriate compensation payments and, in addition, shares the surplus utility obtained by rationalisation. The utility sharing is studied in three different cases. First, it is assumed that the companies do not threaten each other, nor will they ally against each other. The second case allows threats between the firms. The third case is the most general, allowing threats and coalitions inside the enterprise network. In the next section, we present a numerical example.

4 Numerical Application of Innovation Incentives

4.1 Initiation

This section applies the results of Section 2.7 to a network that consists of two suppliers and a client (Figure 11). Let us denote the two suppliers by indices 1 and 2 and the client by index 3. The network manufactures a product, which is being sold to end-customers by the client. The suppliers both have a vital role in the network, supplying the client with certain components of the final product, for which the client pays the suppliers a fixed payment per each component.

Among other duties, supplier 1 carries out a job that, according to cost accounting, involves the expenses of 800 € per each final product manufactured. In negotiations within the network, it has become clear that supplier 2 could perform the same job with costs of only 200 €. That is, by transferring the work from supplier 1 to supplier 2 the network could save up to 600 € per final product. This rationalisation manoeuvre, however, necessitates a change in the fixed payments (Δp_i's) that the client effects to the suppliers. By this simple example, we shall illustrate the calculation of the different reallocations, which are suggested in the Section 2.7.

Following the notation introduced in Section 2.7, we have $N = \{1, 2, 3\}$ and the set of suppliers that are involved in the work transfer process are $N_w = \{1, 2\}$.

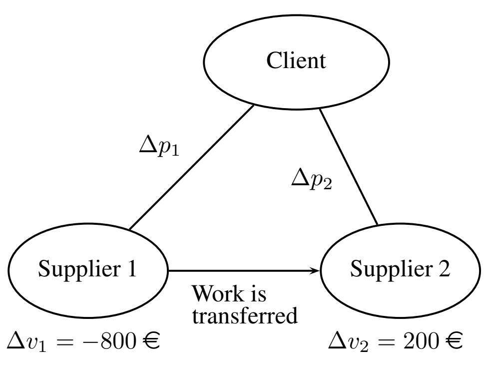

Figure 11. Enterprise Network of the Example.

The changes in the suppliers' costs are

$$\Delta v_1 = -800 \text{ €}$$
$$\Delta v_2 = 200 \text{ €},$$

and the total benefit of the work transfer is (from 27)

$$\pi_{\max} = -\sum_{i \in N_w} \Delta v_i = -(-800 + 200) \text{ €} = 600 \text{ €}.$$

It is noteworthy that, when no threats exist, in the determination of Δp_i's it is sufficient to know the values of Δv_i's; no additional information is needed. In the following Section 4.2, we shall calculate the egalitarian solution with no threats (cf. Section 3.4). Thereafter, in Section 4.3, we shall consider the use of threats and find Nash's relative threats solution (cf. Section 3.6). Finally, in Section 4.4, we shall calculate the allocation according to the modified Shapley value (cf. Section 3.7.5). Section 4.5 compares the results of the different solution concepts.

4.2 Egalitarian Solution without Threats

The egalitarian solution yields the reallocation calculated from the equation (24):

$$\Delta p_1 = \Delta v_1 + \frac{1}{|N_w| + 1} \cdot \pi_{max} = -800 \text{ €} + \frac{1}{2+1} \cdot 600 \text{ €} = -600 \text{ €},$$
$$\Delta p_2 = \Delta v_2 + \frac{1}{|N_w| + 1} \cdot \pi_{max} = 200 \text{ €} + \frac{1}{2+1} \cdot 600 \text{ €} = 400 \text{ €}.$$

That is to say, the supplier 1, whose work load decreases, incurs a fall of -600 € in the fixed payment from the client. Supplier 2, for one, receives a supplement of 400 € for the increased work load. The corresponding utilities, i.e. changes in profit, become (by 11)

$$\pi_1^* = \Delta p_1 - \Delta v_1 = -600 \text{ €} + 800 \text{ €} = 200 \text{ €},$$
$$\pi_2^* = \Delta p_2 - \Delta v_2 = 400 \text{ €} - 200 \text{ €} = 200 \text{ €}.$$

The utility to the client is (by 12)

$$\pi_3^* = -\sum_{i \in N_w} \Delta p_i = -(-600 + 400) \text{ €} = 200 \text{ €},$$

Thereby, in the egalitarian solution, all the participants' utilities are equal, exactly as should be the case by (21).

4.3 Relative Threats Solution

The relative threats solution enables the use of threats, i.e. such actions that can harm a network company if committed by another company of the network. To illustrate how threats can affect the reallocation of the payments, let us assume that the client can terminate the contract with the supplier 1. Assume further, that the client can easily find a substitute supplier, whereas for the supplier 1 it is difficult to find a new customer. Hence, if the contract is terminated, the losses to supplier 1 are valued at $\tau_1 = -150$ €, proportioned to the income of supplier 1 from the present client. The client would not suffer any losses from the termination of the contract ($\tau_3(1) = 0$ €). Under these circumstances, the client possesses a credible threat against supplier 1.

The utilities according to the relative threats approach are solved from the system of linear equations (37)-(38), which, in this example, consists of three equations:

$$
\begin{array}{lrcl}
\text{(i)} & \pi_1 - \tau_1 = \pi_2 & & \pi_1 + 150\,€ = \pi_2 \\
\text{(ii)} & \pi_2 = \pi_3 - \tau_3(1) & \Longrightarrow & \pi_2 = \pi_3 \\
\text{(iii)} & \pi_1 + \pi_2 + \pi_3 = \pi_{\max} & & \pi_1 + \pi_2 + \pi_3 = 600\,€\,.
\end{array}
$$

Inserting (ii) into (i) and (iii) yields

$$
\begin{array}{lrcl}
\text{(iv)} & \pi_1 + 150\,€ & = & \pi_3 \\
\text{(v)} & \pi_1 + \pi_3 + \pi_3 & = & 600\,€\,.
\end{array}
$$

Solving equation (iv) for π_1 and inserting it into (v) gives

$$
\text{(vi)} \quad \pi_3 - 150\,€ + \pi_3 + \pi_3 = 600\,€ \quad \Longrightarrow \quad \pi_3^{\mathrm{T}} = 250\,€\,.
$$

From (i) and (ii) we obtain $\pi_1^{\mathrm{T}} = 100$ € and $\pi_2^{\mathrm{T}} = 250$ €, respectively. The changes in the payments to suppliers become (by 11)

$$
\begin{aligned}
\Delta p_1 &= \pi_1^{\mathrm{T}} + \Delta v_1 = -700\,€\,, \\
\Delta p_2 &= \pi_2^{\mathrm{T}} + \Delta v_2 = 450\,€\,.
\end{aligned}
$$

That is to say, in consequence of supplier 1's dependence on the client, supplier 1 loses 100 € in comparison with the egalitarian solution of Section 4.2. The surplus 100 € is divided up evenly among the client and supplier 2.

4.4 Modified Shapley Value

In determining the solution according to the modified Shapley value, we assume the same interdependencies inside the network as in Section 4.3, i.e. the client possesses a credible threat $\tau_1 = -150$ € against supplier 1. Therefore, the set $N_w = \{1, 2\}$ can be divided into two subsets, namely $W_a = \{1\}$ and $W_b = \{2\}$. Since $W_a \neq \emptyset$, the changes in payments are calculated from (76):

$$\begin{aligned}
\Delta p_1 &= \Delta v_1 + \frac{\pi_{max}}{|N_w| + 1} - \frac{\tau_3(1)}{2} + \frac{\tau_1}{2} \\
&= -800 \text{ €} + \frac{600 \text{ €}}{2+1} - \frac{0 \text{ €}}{2} - \frac{150 \text{ €}}{2} = -675 \text{ €}, \\
\Delta p_2 &= \Delta v_2 + \frac{\pi_{max}}{|N_w| + 1} \\
&= 200 \text{ €} + \frac{600 \text{ €}}{2+1} = 400 \text{ €}.
\end{aligned}$$

With this reallocation of the payments, the utility is shared in the network as follows (by 11 and 12):

$$\begin{aligned}
\pi_1^{\mathrm{S}} &= \Delta p_1 - \Delta v_1 = -675 \text{ €} + 800 \text{ €} = 125 \text{ €}, \\
\pi_2^{\mathrm{S}} &= \Delta p_2 - \Delta v_2 = 400 \text{ €} - 200 \text{ €} = 200 \text{ €}, \\
\pi_3^{\mathrm{S}} &= -\sum_{i \in N_w} \Delta p_i = -(-675 + 400) \text{ €} = 275 \text{ €}.
\end{aligned}$$

4.5 Comparing Different Solution Concepts

Table 1 compares the results of the previous three Sections 4.2, 4.3, and 4.4. The results are depicted in Figure 12. As can be seen, the egalitarian solution π^* yields an equal payoff of 200 € to each company of the network (recall that a company's utility is measured as the change in its profit). This is due to the fact that the egalitarian solution was constructed by defining that the players' utilities should be equal.

The relative threats solution π^{T} penalises the supplier 1 for his dependency in respect of the client. Thus, supplier 1's utility in π^{T} diminishes from π^* proportionally to his losses in a situation of disagreement. The thereby released utility (100 €) is balanced between the client and supplier 2, who would not come to any harm, if the work transfer were rejected.

Table 1. Comparison of the Different Solution Concepts.

Company	π^* (€)	π^{T} (€)	π^{S} (€)
1	200	100	125
2	200	250	200
3	200	250	275

Also the solution according to the modified Shapley value π^{S} takes the dependent situation of supplier 1 into account. However, the only participant who benefits from the weakness of supplier 1 is the one who has the potency to execute the threat, i.e. the client. Again, supplier 1 suffers a loss of utility (-75 €), which is transferred to the client in its entirety. The transferred utility is proportional to the strength of the client's threat, but it is smaller in amount than in the relative threats solution. Also the client is better off in π^{S} than in π^{T}, because the third party, i.e. supplier 2, does not benefit from the client's threat against supplier 1.

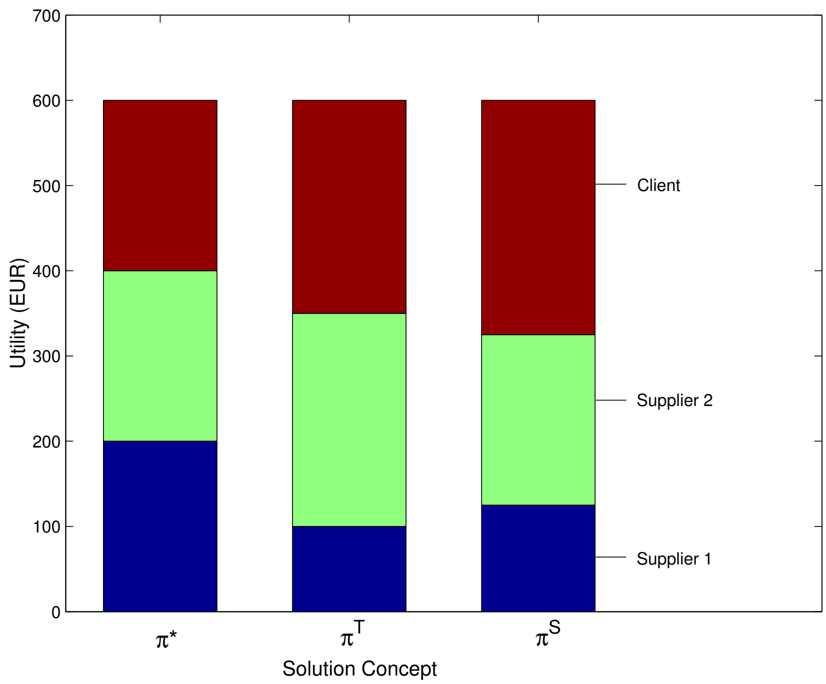

Figure 12. Comparison of the Different Solution Concepts.

It is common to all the three solution concepts that an efficiency-improving arrangement leads to a win-win situation among the companies that are involved in the arrangement. When this is a well-known fact within a network, then the *companies of the network are willing to implement efficiency-improving ideas*. Hence, the network *takes advantage of an innovative atmosphere* inside the network and thus the *competitive advantage of the network sharpens*.

5 Discussion and Conclusions

5.1 Predetermined Innovation Incentives

The ability to improve efficiency via networking is an important advantage, which should be taken into consideration in enterprises. Since rationalisation usually entails potential for win-win situations among the participating companies, the mechanisms of networking induce the firms to implement efficiency-improving ideas. When such an incentive mechanism is inbuilt in the contracts among the network companies, then the atmosphere of the network is innovative and the network is able to improve its competitiveness (Bidault et al. 1998).

The theoretical basis for the incentive mechanisms can be formed, for instance, by a game theoretic study. The main contribution of this paper is a game theoretic model that defines financial incentives for the suppliers of a demand supply network in order to bring forth ideas of rationalising the manufacturing process. Section 2.7 presents three solution concepts, the characteristics of which are illustrated by an example case in Section 3.8. The solution concepts are quantitatively compared and discussed in Section 4.5. Here, we discuss the solution concepts for the implementation of the innovation incentive.

The *egalitarian solution*, referred to as π^*, reflects complete cooperation between the companies, seeing that the utility is shared equally among the parties involved. Hence, we suggest that π^* is to be used when the firms have no intention of harming each other. This could be the case with affiliated companies, for instance. Apart from this, the *relative threats solution* (π^{T}) includes the possibility that the firms may make threats against each other. Threatening may occur for instance between supplier and client, between brand owner and retailer, or in horizontal networks, in which cases π^{T} is a suitable solution. In addition to π^{T}, the solution according to the *modified Shapley value* (π^{S}) takes into account coalition formation. Consequently, π^{S} can be used if the firms have hostile attitudes against each other, which is the situation in e.g. horizontal competition.

Generally, the major advantages of the use of the model are:

1. A win-win situation emerges if a rationalisation manoeuvre is carried out.

2. The information requirements of the companies are minimum.

3. The companies need not exchange confidential information.

4. Cheating is non-profitable.

In the following, we discuss points 1-4.

In the model, the utility function of a network company is defined as the *change in profit*. Therefore, if there is no change in a firm's profit – or the manufacturing process remains unchanged – then the utility of the firm remains zero. However, the rationalisation of the manufacturing process, by definition, produces surplus utility. Sharing the total utility among the network enhances the profit of the companies, hence creating a win-win situation.

In consequence of the form of the utility function, the only information needed is the network companies' cost changes due to the rationalisation manoeuvre. In order for the rationalisation manoeuvre to be feasible, the sum of the cost changes has to be negative i.e. the amount of decreasing costs exceeds the amount of increasing costs. It seems natural that the total utility obtained by the efficiency-improving arrangement is defined to be the sum of the cost changes. Therefore, each of the solution concepts presented in Section 2.7 defines the innovation incentive as a share of the total utility, which can be easily calculated as the sum of the cost changes.

A critical assumption in the model is that the network companies agree to reveal information on their own cost changes due to the rationalisation arrangement. We motivate the assumption by two arguments. First, the amount of cost information to be revealed is normally limited to a single operation that a network company performs among its other duties. That is, the information is only a piece of the total cost information of the company and, hence, is not likely to cause harm to the company in the hands of another firm. Secondly, the information need only be exposed to one agent, who is able to determine the new allocation of payments inside the network. The agent could be, for instance, the client of the network, an independent consultant, or even a computer program. Except for the agent who calculates the payment reallocation, an individual participant of the rationalisation process accesses only the information on his own costs and the total sum of the other participants' cost changes, which an individual can calculate using the information on his own payment reallocation. In consequence, there is no fundamental risk[3] that confidential information leaks to a third party who would benefit from

[3] A comprehensive risk analysis requires a probabilistic approach and is left beyond the scope of this study. For risk management in network economy, see e.g. Hallikas et al. (2001).

the information at the expense of the companies of the network. Thus, we may assume that the firms have no restrictions to share the necessary information.

Cheating, in this case, may occur in two ways. The companies involved in the rationalisation process may distort their cost changes in order to obtain greater compensation from the rationalisation than would be appropriate. However, this form of cheating quickly becomes non-profitable, since it reduces the total utility of the rationalisation arrangement, thus making the arrangement itself seem infeasible to other companies. The other possibility for cheating occurs for the firm that ultimately effects the payments to the other companies. The firm may decide not to pay the predetermined compensation payments, which, self-evidently, causes massive mistrust between the network companies. The prospect of such extreme behaviour is beyond the scope of this study.

The model cannot adapt to every possible situation. We have assumed that the game is played with complete and perfect information, i.e. the utility functions and the previous actions of the participating companies are common knowledge to all participating companies. This, however, need not be the case. Hence, a target for development could be to extend the model to contain uncertainties of, for instance, cost changes and disagreement outcomes. Harsanyi (1967-68) conceptualised games with incomplete information by introducing *games in Bayesian form*. Thereafter, the theory of Bayesian games has been developed to a sophisticated level by numerous researchers. Consequently, the (mathematical) means for the analysis of games with uncertainties exist.

5.2 Summary and Conclusions

In this paper, we have studied the potential of game theoretic modelling of enterprise networks. We have presented a stepwise guideline to the building of game theoretic models that are utilised to support decision-making in the business-network environment. We have also constructed and analysed a game theoretic model that serves as a tool for determining innovation incentives in enterprise networks. In addition, our model provides rules for sharing utility among the companies of a network.

Our postulate that *game theory is a suitable machinery for formal modelling of enterprise networks* was found reasonable. Game theoretic analysis supports analytical decision-making in situations that involve multiple decision-makers who may have (partially) conflicting interests. Consequently, game theoretic models can be exploited to evaluate the possibilities of networking and to find out the premises for successful cooperation. For instance, game theoretic analysis helps to find win-win situations by networking, to collaboratively improve the cost efficiency of networks, and to improve the competitive advantage of networks.

In practice, promising models are implemented in contracts between firms. For instance, the incentive model presented in this paper could be implemented by proactively contracting the payment reallocation rule in the case of a rationalisation manoeuvre. The person who can take the advantage of mathematical models in decision-making is the firm's partnership manager or a worker reporting directly to the manager. The utilisation of the models requires the capability to approach problems analytically. However, the person does not need to have a mathematical background; a basic understanding of the key concepts will suffice.

Probably the most complex part in mathematical modelling of enterprise networks is the estimation of model parameters, which are usually used in utility functions. The problem arises when immaterial benefits, such as quality, innovativeness or responsiveness, have to be assessed quantitatively. Thus, the modelling of such intangible attributes calls for efficient evaluation mechanisms. The mathematical complexity of game theoretic models is case-specific. However, some general conclusions concerning network-economy models can be drawn. First, if utility can be measured in monetary units, then we may assume that utility is transferable, which simplifies the calculus. Second, a model that includes more than two players has to account for coalition formation, which again may complicate the analysis.

The general frame of game theoretic modelling presented in Section 2.7 systemises the model building and aids industry decision-makers in constructing game theoretic models of their own contemporary networking cases. Our case model presented in Section 2.7 offers several mechanisms for determining innovation incentives and sharing utility in a network. The essential use of the model is to create an innovative and efficiency-improving atmosphere inside an enterprise network. Such conditions lead to win-win situations and thus to an absolute gain in utility among the members of the network.

This paper has brought a new perspective to the discussion on decision-making in relation to enterprise networks. The research area is broad; the following – mostly unexplored – subjects have arisen from discussions with firms:

- appraisal / valuation of services provided to a customer
- risk-, cost- and profit-sharing in a network

- investment-sharing in a network
- R&D project management in a network
- simulation of enterprise networks
- modelling uncertainties by Bayesian games.

For further research, these topics yield interesting opportunities to be modelled as games, in which the players are either part of a horizontal network or in a vendor-customer relationship. Besides game theory, other mathematical methodologies may also provide useful tools for network analysis. For instance, network optimisation, portfolio analysis, real-option theory, and other mechanisms for group decision-making are subjects that could be explored in connection with network economy.

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Appendix A: Calculation of the Egalitarian Incentives

The equations (23) can be written

$$\phi_i + \sum_{i \in N_w} \phi_i = \Delta v_i, \quad \forall \ i \in N_w, \tag{A1}$$

which form the following system of linear equations:

$$\begin{aligned} 2\phi_1 + \phi_2 + \ldots + \phi_{n_w} &= \Delta v_1 \\ \phi_1 + 2\phi_2 + \ldots + \phi_{n_w} &= \Delta v_2 \\ &\vdots \\ \phi_1 + \phi_2 + \ldots + 2\phi_{n_w} &= \Delta v_{n_w}. \end{aligned} \tag{A2}$$

In matrix form this becomes

$$\underbrace{\begin{pmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & & \vdots \\ \vdots & & \ddots & 1 \\ 1 & \cdots & 1 & 2 \end{pmatrix}}_{n_w \times n_w} \underbrace{\begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{n_w} \end{pmatrix}}_{n_w \times 1} = \underbrace{\begin{pmatrix} \Delta v_1 \\ \Delta v_2 \\ \vdots \\ \Delta v_{n_w} \end{pmatrix}}_{n_w \times 1}. \tag{A3}$$

The solution is obtained by inverting the $(n_w \times n_w)$-matrix:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{n_w} \end{pmatrix} = \begin{pmatrix} \frac{n_w}{n_w+1} & -\frac{1}{n_w+1} & \cdots & -\frac{1}{n_w+1} \\ -\frac{1}{n_w+1} & \frac{n_w}{n_w+1} & & \vdots \\ \vdots & & \ddots & -\frac{1}{n_w+1} \\ -\frac{1}{n_w+1} & \cdots & -\frac{1}{n_w+1} & \frac{n_w}{n_w+1} \end{pmatrix} \begin{pmatrix} \Delta v_1 \\ \Delta v_2 \\ \vdots \\ \Delta v_{n_w} \end{pmatrix}, \tag{A4}$$

which uniquely defines the ϕ_i's as presented in (24).

Appendix B: Calculation of the Incentives with Threats

The system (37)-(38) is equivalent to

$$\begin{aligned} \pi_1 - \tau_1 &= \pi_2 - \tau_2 \\ \pi_2 - \tau_2 &= \pi_3 - \tau_3 \\ &\vdots \\ \pi_{n_w} - \tau_{n_w} &= \pi_n - \tau_n \\ \sum_{i \in N_w \cup \{n\}} \pi_i &\overset{(27)}{=} \pi_{\max}. \end{aligned} \tag{B1}$$

By moving the π_i's to the left hand side and the τ_i's to the right hand side we obtain

$$\begin{aligned} \pi_1 - \pi_2 &= \tau_1 - \tau_2 \\ \pi_2 - \pi_3 &= \tau_2 - \tau_3 \\ &\vdots \\ \pi_{n_w} - \pi_n &= \tau_{n_w} - \tau_n \\ \sum_{i \in N_w \cup \{n\}} \pi_i &\overset{(27)}{=} \pi_{\max}. \end{aligned} \tag{B2}$$

Writing this in matrix form yields

$$\underbrace{\begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \\ 1 & 1 & \cdots & 1 & 1 \end{pmatrix}}_{(n_w+1)\times(n_w+1)} \underbrace{\begin{pmatrix} \pi_1 \\ \vdots \\ \pi_{n_w} \\ \pi_n \end{pmatrix}}_{(n_w+1)\times 1} = \underbrace{\begin{pmatrix} \tau_1 - \tau_2 \\ \tau_2 - \tau_3 \\ \vdots \\ \tau_{n_w} - \tau_n \end{pmatrix}}_{(n_w+1)\times 1}. \tag{B3}$$

Clearly, the $(n_w + 1) \times (n_w + 1)$-sized coefficient matrix is non-singular and, hence, can be inverted. The unique solution of the system becomes

$$\begin{pmatrix} \pi_1 \\ \vdots \\ \pi_{n_w} \\ \pi_n \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \\ 1 & 1 & \cdots & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \tau_1 - \tau_2 \\ \tau_2 - \tau_3 \\ \vdots \\ \tau_{n_w} - \tau_n \end{pmatrix}. \tag{B4}$$

Appendix C: Calculation of the Modified Shapley Value for the Client

$$\varphi_n = \frac{\pi_{\max} - \sum_{i \in W_a} \tau_i}{|N_w| + 1} + \underbrace{\sum_{k=1}^{|N_w|} \frac{1}{|N_w|+1} \left[\sum_{i \in W_a} \left(\frac{k}{|N_w|} \cdot \tau_n(i) - \frac{|N_w| - k}{|N_w|} \cdot \tau_i \right) \right]}_{=*} \tag{C1}$$

$$* = \frac{1}{|N_w| \cdot (|N_w|+1)} \cdot \underbrace{\sum_{k=1}^{|N_w|} \sum_{i \in W_a} \left[k \cdot \tau_n(i) - (|N_w| - k) \cdot \tau_i \right]}_{=**} \tag{C2}$$

$$\begin{aligned}
** &= \sum_{k=1}^{|N_w|} \sum_{i \in W_a} \left[k \cdot \tau_n(i) - |N_w| \cdot \tau_i + k \cdot \tau_i \right] \\
&= \sum_{k=1}^{|N_w|} \left[k \cdot \left(\sum_{i \in W_a} \tau_n(i) + \sum_{i \in W_a} \tau_i \right) - |N_w| \sum_{i \in W_a} \tau_i \right] \\
&= \left(\sum_{i \in W_a} \tau_n(i) + \sum_{i \in W_a} \tau_i \right) \sum_{k=1}^{|N_w|} k - |N_w| \sum_{i \in W_a} \tau_i \sum_{k=1}^{|N_w|} 1 \\
&= \left(\sum_{i \in W_a} \tau_n(i) + \sum_{i \in W_a} \tau_i \right) \frac{|N_w| \cdot (|N_w|+1)}{2} - |N_w|^2 \sum_{i \in W_a} \tau_i
\end{aligned} \tag{C3}$$

Replacing $**$ in (C2) by (C3) yields

$$\begin{aligned}
* &= \frac{\left(\sum_{i \in W_a} \tau_n(i) + \sum_{i \in W_a} \tau_i \right) (|N_w|+1)}{2(|N_w|+1)} - \frac{2|N_w| \sum_{i \in W_a} \tau_i}{2(|N_w|+1)} \\
&= \frac{(|N_w|+1) \sum_{i \in W_a} \tau_n(i) + (1 - |N_w|) \sum_{i \in W_a} \tau_i}{2(|N_w|+1)}
\end{aligned} \tag{C4}$$

Finally, replacing $*$ in (C1) by (C4) yields

$$\begin{aligned}
\varphi_n &= \frac{\pi_{\max}}{|N_w|+1} + \frac{\sum_{i \in W_a} \tau_n(i)}{2} + \frac{(1 - |N_w|) \sum_{i \in W_a} \tau_i - 2 \sum_{i \in W_a} \tau_i}{2(|N_w|+1)} \\
&= \frac{\pi_{\max}}{|N_w|+1} + \frac{\sum_{i \in W_a} \tau_n(i)}{2} - \frac{\sum_{i \in W_a} \tau_i}{2},
\end{aligned} \tag{C5}$$

which is the desired result (72).

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Innovation Incentives in Enterprise Networks
A Game Theoretic Approach

Abstract

This paper explores the applicability of game theory to the modelling of enterprise networks. Although these networks have traditionally been studied by the qualitative methods of industrial management, the utilisation of game theory seems to provide new tools and solution concepts for studying them. The paper reviews earlier game theoretic studies on inter-firm cooperation and presents a general stepwise pattern for game theoretic modelling of network economy. In addition, the paper constructs a game theoretic model for studying possibilities for creating innovative incentives in an enterprise network.

Inter-firm cooperation is characterised by the interaction of several decision-makers where, on the one hand, the network companies seek joint gains by networking, but, on the other hand, individual companies have their own objectives which may be in partial conflict with those of other companies. Here, game theory provides tools for the formal analysis of situations where multiple decision-makers may have partially conflicting interests, but cooperation between them is allowed.

The determination of innovation incentives in enterprise networks is studied through an application of game theoretic modelling. An example from the boat-building industry is presented to illustrate the relevance of innovation incentives in enterprise networks. Specifically, three different equilibrium concepts are applied to determine innovation incentives under different circumstances. The proposed model helps award innovations that improve the efficiency of the network. In addition, the efficiency-improving arrangements can be implemented so that none of the network companies has to suffer. Consequently, the enterprise network becomes innovative and the network companies need not fear their own losses when the efficiency-improving arrangements are implemented. The model also helps share the surplus utility gained through the innovation among the companies of the network.

Keywords
network economy, enterprise networks, game theory, innovation management, demand supply chain management

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This paper studies the determination of innovation incentives in enterprise networks through an application of game theoretic modelling. Here, game theory provides tools for the formal analysis of situations where multiple decision-makers may have partially conflicting interests, but cooperation between them is allowed. An example from the boat-building industry is presented to illustrate the relevance of innovation incentives in enterprise networks. Specifically, three different equilibrium concepts are applied to determine innovation incentives under different circumstances. The proposed model helps award innovations that improve the efficiency of the network. In addition, the efficiency-improving arrangements can be implemented so that none of the network companies has to suffer. Consequently, the enterprise network becomes innovative and the network companies need not fear their own losses when the efficiency-improving arrangements are implemented. The model also helps share the surplus utility gained through the innovation among the companies of the network.

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